Probabilistic Resilience Analysis of Urban Road Network Considering Wind Damage to Street Trees via Dimension-Reduced Probability Density Evolution Equation (DR-PDEE)

Yi Luo
Institute for Risk & Reliability, Leibniz University Hannover, Hannover 30167, Germany
Email: yi.luo@irz.uni-hannover.de

De-Cheng Feng
School of Civil Engineering, Southeast University, Nanjing 211189, China
Email: dcfeng@seu.edu.cn

Xiao-Qiu Ai
Shanghai Institute of Disaster Prevention and Relief, Shanghai 200092, China
Email: aixiaoqiu@tongji.edu.cn

Michael Beer
Institute for Risk & Reliability, Leibniz University Hannover, Hannover 30167, Germany
Department of Civil & Environmental Engineering, University of Liverpool, Liverpool L69 3GH, UK
International Joint Research Center for Engineering Reliability & Stochastic Mechanics & International Joint Research Center for Resilient Infrastructure, Tongji University, Shanghai 200092, China
Email: beer@irz.uni-hannover.de

Abstract: A probabilistic framework to quantify the resilience of urban road networks considering the wind blowdown of urban trees is proposed. Various properties of street patterns, including length, speed class, and width, as well as the failure probability due to tree-blowdown are considered. The resilience is considered as the mean normalized efficiency during the reconstruction period, and a reconstruction strategy based on the edge betweenness and estimated edge recovering time is employed. The probability density function (PDF) of the estimated resilience under different wind speed is computed by using the dimension-reduced probability density evolution equation (DR-PDEE). The road network of the Inner-Ring area of Yangpu district, Shanghai, China is considered as an example.

Keywords: Probabilistic resilience; urban street network; wind risks; tree blowdown; Dimension-reduced probability density evolution equation (DR-PDEE).

1. Introduction

Estimation of the resilience of urban systems against various attacks has been a hotspot of research in recent years (Bruneau et al. 2003, Cavallaro et al. 2014, Gao et al. 2016). A key challenge in resilience assessment of urban infrastructural systems is the presence of significant uncertainties. Hazard intensity, component vulnerability, post-disaster accessibility, resource availability, and restoration duration may all vary randomly. Accordingly, probabilistic resilience assessment has received increasing attention (Koliou et al. 2020). For example, Kameshwar et al. (2019) developed a probabilistic decision-support framework for community resilience by incorporating multi-hazard effects, infrastructure interdependencies, and resilience goals within a Bayesian-network formulation. Taghizadeh et al. (2023) proposed a probabilistic framework for evaluating transportation-system seismic resilience during emergency medical response by integrating hazard, risk, agent-based emergency response, and network congestion modules. Rincon and Padgett (2024) used probabilistic fragility models to propagate uncertainty from component-level damage to network-level resilience indicators for bridges and transportation-related infrastructure systems. Cao and Feng (2025) proposed probabilistic resilience assessment frameworks for engineering structures by explicitly incorporating uncertainties in functionality, recovery time, and recovery duration. These advances demonstrate the importance of representing resilience as a probabilistic quantity rather than a deterministic index.

In this study, we focus on the road network, one of the most important components of a city, and analyze its resilience considering the wind-blowdown of street trees. The road network of the Inner-Ring-area of Yangpu district, Shanghai city in China is used as an example to illustrate the methods and procedures. Specifically, the probability of road closure due to fallen trees are already given by a developed model (Luo & Ai 2022). The intention of this project is to quantify the resilience of streets in a city as a whole network after wind attacks. The classical definition of the resilience based on the network efficiency is employed (Bruneau et al. 2003). By taking into consideration the road width and speed limit, the transportation efficiency on the basis of travel-time budget is obtained. Then, the resilience of the network at a given damage state can be readily calculated with the assistance of a reconstruction strategy determined jointly by the edge betweenness and edge reconstruction time.

Since the wind damage of roads is a random event, the statistics of resilience are desired. In this context, the dimension-reduced probability density evolution equation (DR-PDEE) (Lyu & Chen 2022) is applied and developed for this problem by constructing a "resilience process". The reliability and accuracy of the method is verified by pertinent Monte Carlo simulations. The probability density function (PDF) and other statistical quantities of network resilience under different wind speed are obtained and discussed.

2. Resilience of Street Network

2.1 BASIS INFORMATION OF THE NETWORK

A road network with 181 nodes and 298 links, as shown in Figure 1, is considered as an example to demonstrate the method and procedures proposed in this project. The inherent road information required for the network analysis including width (b_i), maximum speed (v_i), edge length (L_i), and node coordinates (x_i, y_i) are predefined according to the real situation, and assigned to each edge.

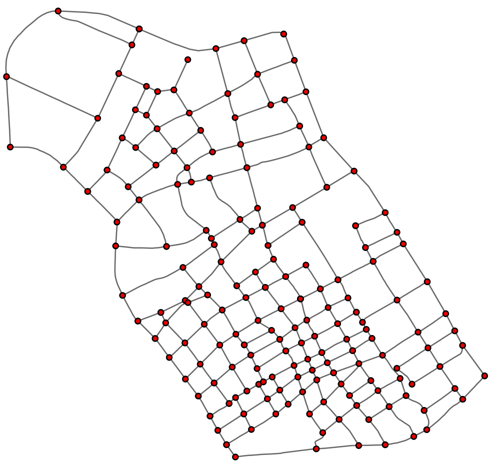

Figure 1. Network Topology.

The dominant tree species and tree height of each road should also be predetermined, but since the data is not available yet, the tree species and heights are assigned according to assumed distributions in this illustrative case study. Each edge is assumed to comprise only one tree species and one dominant tree height. Based on a wind fragility assessment model of trees, the failure probability of a single tree P_{T_i} can be obtained, and the edge failure probability is given by

$$P_i = 1-\left(1-P_{T_i}\right)^{N_i/N_c}. \tag{1}$$

In this equation, P_i is the failure probability of edge i; N_i is the number of trees in edge i, and is determined by $N_i = \{L_i/6.5\}\times 2$, where $\{\cdot\}$ denotes rounding-numbers, L_i is the length of the edge in meters; N_c is the number of trees whose performance are considered to be strongly connected. In this example, N_c is taken as 20.

A dangerous index (DI_i) is also allocated to each edge according to their dominant tree species and height. It is a number between 0 and 4, and will work as an essential factor for the reconstruction strategy in following calculation of network resilience.

2.2 TRAVEL-TIME BASED EFFICIENCY

Since in transportation networks, the travel time, instead of the physical distance, is a more logical measure for the "shortest path", with respect to human drivers' intuition (Lämmer et al. 2006), the global efficiency of a street network is determined based on the travel time budget. That is,

$$Eff = \frac{1}{N\left(N-1\right)}\sum_{i,j\in\aleph,i\neq j}\frac{\tau_{ij}^{\mathrm{eucl}}}{\tau_{ij}}. \tag{2}$$

In this equation, N is the number of nodes; $\aleph$ denotes the set of all nodes. τ_{ij} denotes the shortest path between node i and j regarding the travel-time budget. The travel-time budget of each edge is calculated as

$$\tau_i = \frac{L_i}{\alpha_i v_i}, \tag{3}$$

where v_i is the speed limit, which takes value from $[40, 60, 70]$ km/h; α_i is a coefficient representing the influence of the street width to the travel speed, which is 0.7, 1, and 1.5 for narrow, medium, and wide street respectively. Note that the "travel-time budget" used here does not represent any "real" travel time, and it only makes sense in terms of relative value. τ_{ij}^{eucl} is a term to normalize pair-wise efficiency over all possible pairs of nodes, and it is defined as

$$\tau_{ij}^{\text{eucl}} = \frac{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{70 \times 1.5}, \tag{4}$$

such that $\tau_{ij}^{\text{eucl}} / \tau_{ij}$ is always between 0 and 1.

2.3 MEASURE OF RESILIENCE

The classical definition of urban resilience is employed here, that is (Bruneau et al. 2003)

$$R = \frac{\int_{t_1}^{t_2} Y(t)\,\mathrm{d}t}{t_2 - t_1}, \tag{5}$$

where $Y(t)$ is the recovery function, t_1 and t_2 are respectively the starting time and completed time of the recovery process.

The recovery function $Y(t)$ is defined as the ratio between real-time efficiency $Eff(t)$ and the efficiency of the original network Eff_0, namely,

$$Y(t) = Eff(t) / Eff_0 . \tag{6}$$

If no damage occurs, the resilience R is set to be one. Notice that R depends on both the total damage and the reconstruction strategy. Because the intention of the project is not to compare the performance of different recovery strategies, but to assess the performance of the network subjected to different wind speed, only one reconstruction strategy is employed and will be soon introduced.

2.4 RECONSTRUCTION STRATEGY

In this project, a relative inflexible scheme is employed to simulate the reconstruction procedure for all the damaged networks. That is, the damaged edges are repaired one by one, and the sequence of the edges to be recovered has been pre-determined based on their relative "importance" and the order of difficulty to repair.

The "importance" is measured by travel-time-based edge betweenness, namely:

$$B_i = \sum_{s,t \in \aleph} \frac{n_{st}(i)}{N_{st}}, \tag{7}$$

where $n_{st}(i)$ is the number of shortest paths from node s to t that pass through edge i, N_{st} is the total number of shortest paths from node s to t.

The difficulty to repair is quantified by the estimated reconstruction time, which is a pseudo variable specifically constructed in this project. It is calculated as

$$RT_i = \ln\left(\frac{DI_i \cdot L_i}{100} + 1\right) \Big/ 10 . \tag{8}$$

Notice that similar as the travel-time budge τ_i, the reconstruction time RT_i only makes sense with respect to relative value, as well.

The order of edge reconstruction is determined by the order of B_i / RT_i, so that the edges of higher "importance" and with less reconstruction effort will be recovered earlier.

After determined the reconstruction strategy, the resilience can be calculated accordingly. However, since the total damage is a random event, the resilience at a given wind speed should be a random variable. Therefore, its statics, such as mean value, probability density function, and quantiles, are of concern. They will be analyzed in the next section.

3. Statistical Analysis of Resilience Based on DR-PDEE

In this section, the statistics of network resilience at different wind speed will be analyzed. Since the probability density function (PDF) is much more informative than statistical moments such as mean and standard deviation, it is the object we desired. However, using crude Monte Carlo Simulation to obtain the PDF is too time-consuming, even for the small network considered in this simple example. Therefore, the recently developed dimension-reduced probability density evolution equation (DR-PDEE) (Lyu & Chen 2022) technique is further developed to tackle the problem more efficiently.

3.1 FORMULATION OF DR-PDEE

For any one-dimensional continuous stochastic process $X(t)$ (the continuity of a stochastic process is defined by the Lindeberg's condition (Gardiner 2004)), its PDF $p_X(x,t)$ satisfies the following equation:

$$\frac{\partial p_X}{\partial t} = -\frac{\partial\left[a^{(\mathrm{int})}(x,t)\, p_X\right]}{\partial x} + \frac{1}{2}\frac{\partial^2\left[b^{(\mathrm{int})}(x,t)\, p_X\right]}{\partial x^2}, \tag{9}$$

where

$$a^{(\mathrm{int})}(x,t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathrm{E}\left[\Delta X \middle| X(t) = x\right], \tag{10}$$

$$b^{(\mathrm{int})}(x,t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \mathrm{E}\left[(\Delta X)^2 \middle| X(t) = x\right], \tag{11}$$

and $\Delta X = X(t+\Delta t) - X(t)$. Equation (9) is just the DR-PDEE for general continues random process (Lyu & Chen 2022). $a^{(\mathrm{int})}(x,t)$ and $b^{(\mathrm{int})}(x,t)$ is referred to as the intrinsic drift and diffusion coefficients, respectively.

3.2 DR-PDEE for determining the PDF of Resilience

The DR-PDEE is suitable for determine the time-varying probability of a stochastic process. The Resilience $R(\boldsymbol{\Theta})$, however, is a random variable determined by several random parameters $\boldsymbol{\Theta}$. Specifically, in this example, the random parameters are the performance of each edge, that is, $\boldsymbol{\Theta} = [\Theta_1, \Theta_2, \cdots, \Theta_m]$, where m is the total number of edges, and Θ_i, $i = 1, \cdots, m$ are independent 0-1 variables. In this context, a process based on Equation (5) is constructed, that is

$$R(\boldsymbol{\Theta}, t) = \begin{cases} \dfrac{\int_0^t Y(\boldsymbol{\Theta}, \tau) \mathrm{d}\tau}{T_r(\boldsymbol{\Theta})}, & t < T_r(\boldsymbol{\Theta}) \\ \dfrac{\int_0^{T_r(\Theta)} Y(\boldsymbol{\Theta}, \tau) \mathrm{d}\tau}{T_r(\boldsymbol{\Theta})}, & t \geq T_r(\boldsymbol{\Theta}) \end{cases}. \tag{12}$$

In this equation, $T_r(\boldsymbol{\Theta})$ denotes the total reconstruction time, and $R(\boldsymbol{\Theta}, t)$ is defined as the resilience process.

Considering the DR-PDEE of $R(\boldsymbol{\Theta}, t)$, one obtains

$$a^{(\mathrm{int})}(r, t) = \mathrm{E}\left[\dot{R}(t) \middle| R(t) = r\right], \tag{13}$$

$$b^{(\mathrm{int})}(r, t) = \lim_{\Delta t \to 0} O(\Delta t) = 0, \tag{14}$$

and

$$\dot{R}(\boldsymbol{\Theta}, t) = \begin{cases} \dfrac{Y(\boldsymbol{\Theta}, \tau)}{T_r(\boldsymbol{\Theta})}, & t < RT(\boldsymbol{\Theta}) \\ 0, & t \geq RT(\boldsymbol{\Theta}) \end{cases}. \tag{15}$$

Consequently, the DR-PDEE in the form of equation (9) can be cast as

$$\frac{\partial p_R(r, t)}{\partial t} = -\frac{\partial\left[a^{(\mathrm{int})}(r, t) p_R(r, t)\right]}{\partial r}. \tag{16}$$

Equation (16) has a form similar to Liouville equation, and can be easily solved numerically. Notice that the intrinsic drift coefficient $a^{(\mathrm{int})}(r, t)$ is still implicit. Since its nature is the conditional mean function, it is estimated from samples at each time instant in the following analysis by using the locally weighted smoothing scatterplots (Lyu & Chen 2021). Figure 2 shows a typical example of the fitting result.

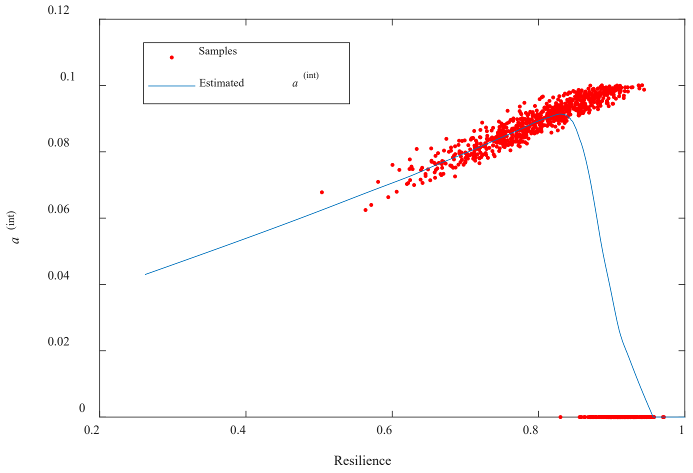

Figure 2. Typical fitting result of $a^{(\text{int})}(r,t)$.

3.3 NUMERICAL RESULTS

In this section, the PDFs of resilience are solved by the DR-PDEE when the mean wind speed at 10m heigh U_{10} of the region considered is set to be 10 m/s, 15 m/s, 20 m/s, …, 40 m/s respectively.

Before proceeding to the final results, the reliability of the DR-PDEE is verified by comparing the PDF obtained by the DR-PDEE when $U_{10} = 20\ \text{m/s}$ with pertinent Monte Carlo simulations. The results are shown in Figure 3. 1000 samples are used for estimating the $a^{(\text{int})}(r,t)$ of the DR-PDEE. It is obvious that the PDF obtained by the DR-PDEE agrees well with the results of MCS with 10^5 samples, and is much better than the PDF directly obtained from the 1000 samples.

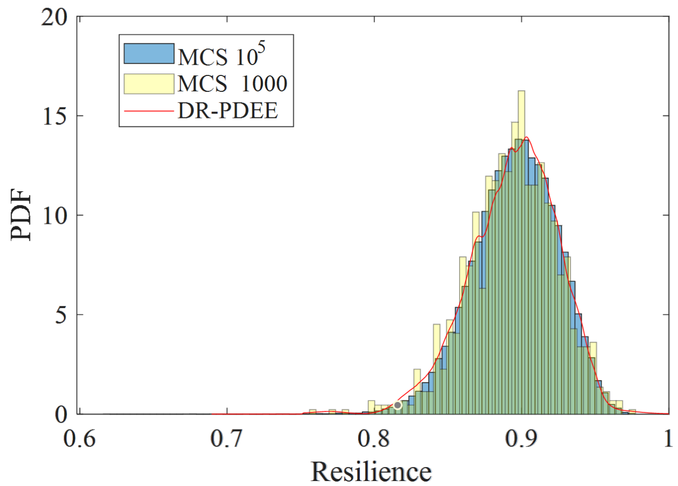

Figure 3. Comparison of the PDFs of resilience when $U_{10} = 20$ m/s .

Next, the PDFs under each wind condition are calculated by the DR-PDEE, see Figure 4 and Figure 5. Notice that the peak value of the PDF when $U_{10} = 10$ m/s is much higher than others, so it is plotted individually.

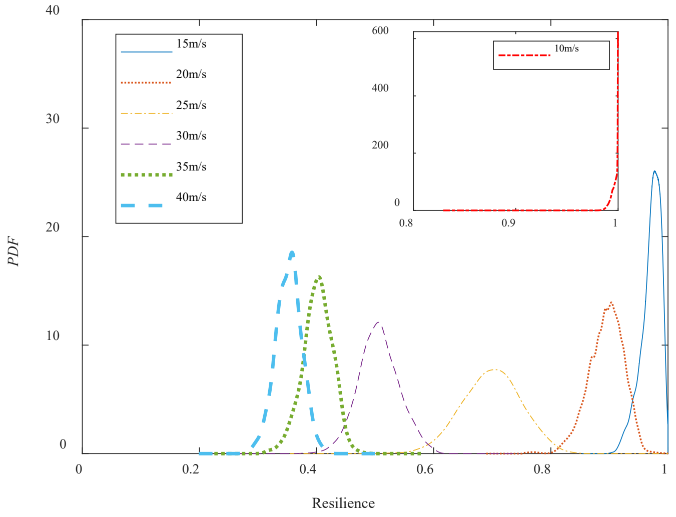

Figure 4. PDFs of resilience at different mean wind speeds.

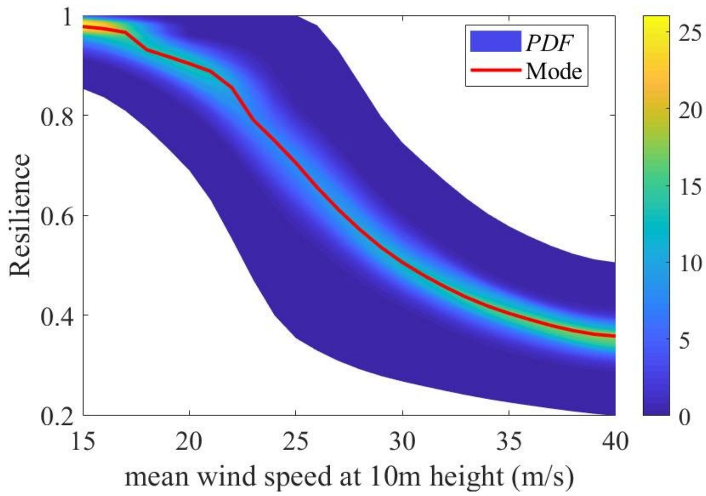

Figure 5. PDFs and modes of resilience at different mean wind speeds.

Except for the PDF results, the mean value and the quantiles may be more intuitive in engineering practice. For instance, the 10% quantile means that with a probability of 90%, or in other word, with a confidence level of 90%, the resilience of the network is higher than that value. Figure 6 shows the means and quantiles of the resilience under different mean wind speeds. It can be seen that the resilience approaches to 1 at $U_{10} = 10$ m/s, and decreases with the increase of U_{10} in general. This is consistent with common sense. From Figure 4 and Figure 6, it is also obvious that the variability of the resilience is large for medium wind speed (around 25 m/s), and is small for very small or large wind.

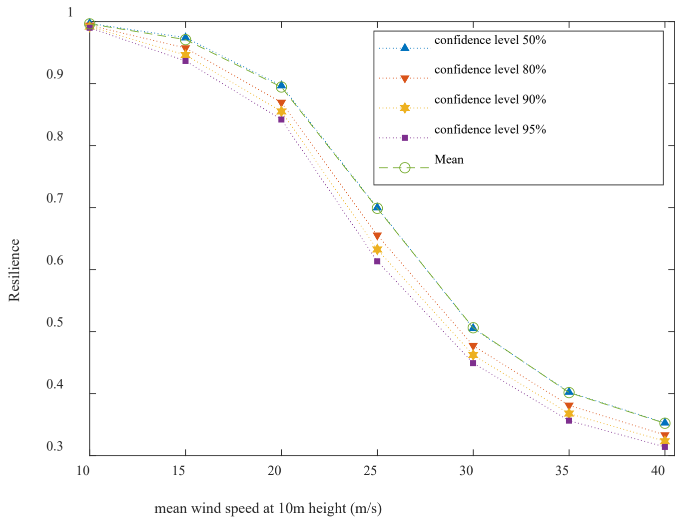

Figure 6. Means and quantiles of resilience at different mean wind speeds.

4. Concluding Remarks

In this study, a procedure to quantify the value and uncertainty of the resilience of urban road network considering the wind damage to street trees is proposed. Several steps have been taken.

(1) The classical definition of resilience is employed to quantify the resilience of the urban road network. When uncertainties of various input sources are considered, the statistics of the resilience indicator can be obtained.

(2) A method based on the dimension-reduced probability density is proposed to calculate various statistics of the resilience efficiently. Comparison with pertinent Monte Carlo simulations illustrates the reliability of the proposed method.

(3) The resilience under different wind speed is quantified and expressed by various means. Specially, the resilience values of different confidence level are obtained.

References

Der Kiureghian, A. & Dakessian, T. 1998. Multiple design points in first and second-order reliability. *Structural Safety* 20(1): 37-49.

Ditlevsen, O. & Madsen, H.O. 1996. *Structural Reliability Methods*. New York: John Wiley & Sons.

Bucher, C. 2009. Computational analysis of randomness in structural mechanics. In D.M. Frangopol (Ed.), *Structures and Infrastructures Book Series*, Vol. 3, London: Taylor & Francis, pp. xxx-xxx.

Bruneau, M. et al. 2003. A Framework to Quantitatively Assess and Enhance the Seismic Resilience of Communities. *Earthquake Spectra,* 19(4):733–752.

Cao, X. Y., & Feng, D. C. (2025). Probabilistic resilience assessment framework of structures considering the combined functionality-recovery-duration uncertainty. *Mechanical Systems and Signal Processing*, *223*, 111856.

Cavallaro, M. et al. 2014. Assessment of Urban Ecosystem Resilience through Hybrid Social-Physical Complex Networks. *Computer-Aided Civil and Infrastructure Engineering,* 29(8):608–625.

Gao, J. Barzel, B. & Barabási A.L. 2016. Universal Resilience Patterns in Complex Networks. *Nature,* 530(7590):307–312.

Gardiner, C.W. 2004. *Handbook of Stochastic Methods*. Berlin Heidelberg: Springer-Verlag.

Kameshwar, S., Cox, D. T., Barbosa, A. R., Farokhnia, K., Park, H., Alam, M. S., & van de Lindt, J. W. (2019). Probabilistic decision-support framework for community resilience: Incorporating multi-hazards, infrastructure interdependencies, and resilience goals in a Bayesian network. *Reliability Engineering & System Safety*, *191*, 106568.

Koliou, M., van de Lindt, J. W., McAllister, T. P., Ellingwood, B. R., Dillard, M., & Cutler, H. (2020). State of the research in community resilience: Progress and challenges. *Sustainable and resilient infrastructure*, *5*(3), 131-151.

Lämmer, S. Gehlsen, B. & Helbing, D. 2006. Scaling Laws in the Spatial Structure of Urban Road Networks. *Physica A: Statistical Mechanics and Its Applications,* 363(1):89–95.

Luo, Y., & Ai, X. (2022). Wind risk assessment of urban street trees based on wind-induced fragility. *Disaster Prevention and Resilience*, *1*(2), N-A.

Lyu, M.Z. & Chen J.B. 2021. First-Passage Reliability of High-Dimensional Nonlinear Systems under Additive Excitation by the Ensemble-Evolving-Based Generalized Density Evolution Equation. *Probabilistic Engineering Mechanics,* 63 (2021):103119

Lyu, M.Z. & Chen J.B. 2022. A unified formalism of the GE-GDEE for generic continuous responses and first-passage reliability analysis of high-dimensional nonlinear systems subjected to non-white noise excitations. *Structural Safety,* 98:102233.

Rincon, R., & Padgett, J. E. (2024). Fragility modeling practices and their implications on risk and resilience analysis: From the structure to the network scale. *Earthquake spectra*, *40*(1), 647-673.

Taghizadeh, M., Mahsuli, M., & Poorzahedy, H. (2023). Probabilistic framework for evaluating the seismic resilience of transportation systems during emergency medical response. *Reliability Engineering & System Safety*, *236*, 109255.