# Three-Dimensional Multivariable Non-Gaussian Random Field Modeling and Simulation of Concrete Constitutive Law


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**Abstract:** The constitutive behavior of concrete exhibits significant randomness and spatial variability, which necessitates a probabilistic description for accurate mechanical analysis. In this study, a comprehensive framework is developed for the unified representation and simulation of four key constitutive parameters (including elastic modulus, compressive strengths, peak compressive strains, and compressive shape parameters) using three-dimensional (3D) multivariate non-Gaussian random fields. The uncertainties are represented at three hierarchical levels: variable randomness (modeled via marginal distributions), inter-variable dependence (modeled via copulas), and spatial variability (captured by correlation functions). To achieve this, two novel concepts, i.e., the bridge function and the transformed correlation function, are introduced, enabling the generation of random fields that rigorously satisfy prescribed marginal distributions, cross-dependencies, and spatial correlations. For efficient simulation, the parsimonious stochastic harmonic function (SHF) is employed. Within this framework, the three-dimensional wavenumber domain is partitioned using a Voronoi tessellation scheme, enabling the spectrum-dependent random wavenumbers to be represented with a significantly reduced number of basic random variables. In addition, a starting-time phase evolution strategy is introduced to further minimize the number of random variables required for characterizing the random phases. As a result, only 28 basic random variables are sufficient to generate high-fidelity realizations of the four-variable dependent fields. The proposed approach provides a practical and computationally efficient tool for stochastic finite element analysis of concrete structures, enabling rigorous assessment of structural performance under material uncertainties.

**Keywords:** Concrete constitutive law; 3D multivariate non-Gaussian random fields; Vine copula; Probabilistic dependency; Spatial variability.

## 1. Introduction

Concrete is one of the most widely used construction materials in civil infrastructure, and the majority of existing engineering structures are constructed from concrete (Chen et al., 2025). Throughout their service life, these structures are subjected to various loading conditions (Ghanem and Spanos, 2003; Li and Chen, 2009), making the accurate assessment of their mechanical behavior and load-carrying capacity of paramount importance. However, uncertainties are inherently present in both materials and loading environments and therefore cannot be neglected in structural analysis and design (Lyu et al., 2024a, 2025b).

As demonstrated by Tao et al. (2020; 2023), concrete constitutive parameters are characterized by substantial randomness, spatial variability, and complex inter-parameter dependence. These three levels of uncertainty must be properly considered to achieve reliable predictions of structural performance and safety. Random fields provide an effective probabilistic framework for characterizing both the inherent randomness and spatial variability of material properties, and have therefore been widely employed for uncertainty modeling in engineering applications (Ghanem and Spanos, 2003; Li and Chen, 2009). However, most concrete constitutive parameters exhibit distinctly non-Gaussian behavior (Tao et al., 2020), rendering Gaussian random field models inadequate for accurate characterization. Moreover, practical concrete structures are inherently three-dimensional, while different constitutive parameters are often statistically dependent (Tao et al., 2020). Therefore, a realistic probabilistic description of concrete material properties necessitates the development of three-dimensional multivariate non-Gaussian random field models capable of simultaneously capturing marginal non-Gaussianity, spatial correlation, and inter-variable dependence.

Random field modeling plays a fundamental role in uncertainty quantification and propagation for concrete structures and serves as a prerequisite for random field simulation. In general, the randomness of individual variables is characterized by their marginal probability distributions (Benjamin and Cornell, 2014), spatial variability is described by autocorrelation functions in the physical domain or power spectral density functions in the frequency domain (Ghanem and Spanos, 2003), and probabilistic dependence among variables is commonly represented using copulas or vine copulas (Tao et al., 2020; Lyu et al., 2024b). Owing to their ability to decouple a multivariate joint probability density function into marginal distributions and copula density functions, copula-based approaches provide a flexible and efficient framework for modeling complex dependence structures (Lyu et al., 2024c).

However, for multivariate non-Gaussian random fields, these three probabilistic descriptions, i.e, marginal distributions, spatial correlations, and inter-variable dependence, are generally incompatible (Liu et al., 2026; Fu et al., 2026). Consequently, conventional simulation methods often require iterative procedures (Liu and Der Kiureghian, 1986; Phoon et al., 2005; Zheng et al., 2023) or auxiliary transformations, such as Hermite polynomial models (Li and Xu, 2024) and the S-transform (Huang and Xu, 2021), to approximately satisfy the prescribed statistical characteristics. To address these challenges, several studies have focused on correlation-based approaches. Lyu et al. (2025a) proposed the bridge function, which establishes a probabilistic relationship among the autocorrelation functions of the target field, the dependent field, and an independent standard Gaussian field. Within this framework, the dependent field can be constructed from the main field and an independent Gaussian field, thereby resolving the incompatibility between prescribed marginal distributions and copula- or vine-copula-based dependence structures. Furthermore, Liu et al. (2026) developed the transformed correlation function, which enables the direct generation of non-Gaussian random fields from standard Gaussian fields while exactly preserving the target marginal distributions and spatial correlations without iterative procedures. Building upon these developments, the present study adopts both the bridge function and the transformed correlation function to establish a probability-compatible framework for the modeling of three-dimensional multivariate non-Gaussian random fields representing concrete constitutive parameters.

Once the probabilistic modeling is completed, the next step is the simulation of random fields based on the established uncertainty models. However, many widely used simulation techniques, such as the spectral representation method (Shinozuka and Deodatis, 1996), require a large number of basic random variables to accurately reproduce the target stochastic characteristics. This limitation becomes particularly severe for three-dimensional random fields, resulting in substantial computational costs and a pronounced curse-of-dimensionality problem.

To address this challenge, a parsimonious stochastic harmonic function (SHF) representation is adopted in this study. The proposed approach incorporates Voronoi wavenumber partitioning (Song et al., 2019), spectrum-dependent random wavenumbers (Chen et al., 2020), and the concept of phase-evolution starting points (Li et al., 2013) into the SHF framework (Chen et al., 2013, 2017). As a result, a three-dimensional random field can be represented with high accuracy using only seven basic random variables, leading to a dramatic reduction in stochastic dimensionality and computational effort. Therefore, once the transformed correlation function corresponding to the target non-Gaussian field is obtained, the parsimonious SHF representation is employed to generate the underlying standard Gaussian field. Subsequently, three-dimensional non-Gaussian random fields of the concrete constitutive parameters are constructed using the proposed probability-compatible transformation framework. The bridge function is employed to establish the probabilistic dependence among the parameter fields.

The remainder of this paper is organized as follows. Section 2 presents the probability-compatible modeling framework for the concrete constitutive parameters. Section 0 introduces the parsimonious SHF representation and the corresponding simulation procedure, followed by numerical investigations to validate the accuracy and efficiency of the proposed approach. Finally, conclusions are drawn in Section 4.

## 2. Probabilistic Modeling of Three-Dimensional Random Fields for Concrete Constitutive Parameters

In this section, the constitutive parameter fields of concrete, including the elastic modulus $E(\boldsymbol{x})$, compressive strength $f_{\mathrm{c}}(\boldsymbol{x})$, peak strain $\varepsilon_{\mathrm{c}}(\boldsymbol{x})$, and the parameter governing the descending branch $\alpha_{\mathrm{c}}(\boldsymbol{x})$, are modeled as three-dimensional random fields. Here, $\boldsymbol{x} = (x_1, x_2, x_3)^{\mathrm{T}}$ denotes the spatial coordinate vector.

### 2.1. Probabilistic Characterization of Three Sources of Uncertainty

The three levels of uncertainty, namely parameter randomness, spatial variability, and inter-parameter dependence, are characterized probabilistically in this study.

First, the marginal distributions of the constitutive parameters are specified based on available experimental data. The elastic modulus $E$ and compressive strength $f_c$ are modeled using lognormal distributions, whereas the peak strain $\varepsilon_c$ and the descending-branch parameter $\alpha_c$ are represented by $\Gamma$ and shifted-$\Gamma$ Gamma distributions, respectively. The corresponding probability density functions (PDFs) are given as follows:

$$\begin{cases} p_E(w) = \dfrac{1}{\sqrt{2\pi}\,\sigma_E w} e^{-\frac{1}{2\sigma_E^2}\ln^2\frac{w}{r_E}}, & \text{for } w \geqslant 0, \\ p_f(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma_f x} e^{-\frac{1}{2\sigma_f^2}\ln^2\frac{x}{r_f}}, & \text{for } x \geqslant 0, \\ p_\varepsilon(y) = \dfrac{y^{k_\varepsilon - 1} e^{-\frac{y}{\theta_\varepsilon}}}{\Gamma(k_\varepsilon)\theta_\varepsilon^{k_\varepsilon}}, & \text{for } y \geqslant 0, \\ p_\alpha(z) = \dfrac{(z-r_\alpha)^{k_\alpha - 1} e^{-\frac{z-r_\alpha}{\theta_\alpha}}}{\Gamma(k_\alpha)\theta_\alpha^{k_\alpha}}, & \text{for } z \geqslant r_\alpha, \end{cases} \tag{1}$$

where $r_E$, $\sigma_E$, $r_f$, $\sigma_f$, $\theta_\varepsilon$, $k_\varepsilon$, $r_\alpha$, $\theta_\alpha$, and $k_\alpha$ denote the distribution parameters; $\Gamma(\cdot)$ is the Γ function.

Based on the study of Tao et al. (2020), probabilistic dependence exists among the parameters $f_c$, $\varepsilon_c$ and $\alpha_c$.To accurately characterize their joint behavior, the probabilistic dependence among $f_c(\boldsymbol{x})$, $\varepsilon_c(\boldsymbol{x})$, and $\alpha_c(\boldsymbol{x})$ must be explicitly modeled. The joint probability density function (PDF) at an arbitrary spatial location $\boldsymbol{x}$ is constructed using a vine copula (Tao et al., 2020), i.e.,

$$\begin{aligned} p_{f\varepsilon\alpha}(x,y,z) = & c_{f\varepsilon|\alpha}\{C_{f|\alpha}[F_{f_c}(x)\,|F_\alpha(z)\,]C_{\varepsilon|\alpha}[F_\varepsilon(y)\,|F_\alpha(z)\,]\} \\ & \cdot c_{f\alpha}[F_f(x),F_\alpha(z)]c_{\varepsilon\alpha}[F_\varepsilon(y),F_\alpha(z)]p_f(x)p_\varepsilon(y)p_\alpha(z), \end{aligned} \tag{2}$$

According to Tao et al. (2020), the dependence structures represented by $c_{f\alpha}(\cdot,\cdot)$, $c_{\varepsilon\alpha}(\cdot,\cdot)$, and $c_{f\varepsilon|\alpha}(\cdot,\cdot)$ are best described by the Frank copula, Gaussian copula, and 90° Clayton copula, respectively. The corresponding copula density functions are expressed as follows

$$\begin{cases} c_{f\alpha}(u,v) = \dfrac{-\theta_{f\alpha}(e^{-\theta_{f\alpha}}-1)e^{-\theta_{f\alpha}(u+v)}}{[e^{-\theta_{f\alpha}}-1+(e^{-\theta_{f\alpha}u}-1)\,(e^{-\theta_{f\alpha}v}-1)]^2}, \\ c_{\varepsilon\alpha}(u,v) = \dfrac{1}{\sqrt{1-\varrho_{\varepsilon\alpha}^2}} e^{-\frac{\varrho_{\varepsilon\alpha}^2[\Phi^{-1}(u)]^2 - 2\varrho_{\varepsilon\alpha}\Phi^{-1}(u)\Phi^{-1}(v) + \varrho_{\varepsilon\alpha}^2[\Phi^{-1}(v)]^2}{2(1-\varrho_{\varepsilon\alpha}^2)}}, \\ c_{f\varepsilon|\alpha}(u,v) = \dfrac{1+\theta_{f\varepsilon}}{(uv)^{1+\theta_{f\varepsilon}}\left(\dfrac{1}{u^{\theta_{f\varepsilon}}}+\dfrac{1}{v^{\theta_{f\varepsilon}}}-1\right)^{2+\frac{1}{\theta_{f\varepsilon}}}}, \end{cases} \tag{3}$$

where $\theta_{f\alpha}$, $\varrho_{\varepsilon\alpha}$, and $\theta_{f\varepsilon}$ denote the copula parameters.

The spatial variability of the constitutive parameters is characterized by the exponential correlation model (Tao and Chen, 2023), i.e.,

$$\rho_E(\boldsymbol{\xi})=e^{-\frac{|\boldsymbol{\xi}|}{l_E}},\ \rho_f(\boldsymbol{\xi})=e^{-\frac{|\boldsymbol{\xi}|}{l_f}},\ \rho_\varepsilon(\boldsymbol{\xi})=e^{-\frac{|\boldsymbol{\xi}|}{l_\varepsilon}},\ \text{and}\ \rho_\alpha(\boldsymbol{\xi})=e^{-\frac{|\boldsymbol{\xi}|}{l_\alpha}}, \tag{4}$$

where $l_E$, $l_f$, $l_\varepsilon$, and $l_\alpha$ denote the correlation lengths. The target statistical parameters for the uncertainty model are summarized in Table ITable I and Table II. With the marginal distributions, copula-based dependence structure, and correlation functions fully specified, the three levels of uncertainty are completely characterized.

Table I. Marginal distribution models and statistical parameters of concrete constitutive parameters.

| Constitutive parameters | Distribution | Parameters of distribution | Mean | COV | Correlation length |
|---|---|---|---|---|---|
| $E_c$ | Log-normal | $r_E=3.66\times10^{10}\text{Pa},\ \sigma_E=0.149$ | $3.7\times10^{10}\text{Pa}$ | 0.15 | $l_E=0.18\ \text{m}$ |
| $f_c$ | Log-normal | $r_f=2.27\times10^{7}\text{Pa},\ \sigma_E=0.149$ | $2.3\times10^{7}\text{Pa}$ | 0.15 | $l_f=0.32\ \text{m}$ |
| $\varepsilon_c$ | $\Gamma$ | $\theta_\varepsilon=2.25\times10^{-5},k_\varepsilon=44.44$ | $1.0\times10^{-3}$ | 0.15 | $l_\varepsilon=0.40\ \text{m}$ |
| $\alpha_c$ | Shifted $\Gamma$ | $r_\alpha=0.3,\theta_\alpha=0.25,k_\alpha=0.79$ | 0.5 | 0.45 | $l_\alpha=0.17\ \text{m}$ |

Table II. Vine copula model for concrete constitutive parameters.

| Constitutive parameters | Copula | Copula parameter s |
|---|---|---|
| $f_c$, $\alpha_c$ | Frank | $\theta_{f\alpha}=4.4$ |
| $\varepsilon_c$, $\alpha_c$ | Gauss | $\varrho_{\varepsilon\alpha}=0.69$ |
| $f_c,\varepsilon_c\|\alpha_c$ | 90 ∘Clayton | $\theta_{f\varepsilon}=0.51$ |

## 2.2. PROBABILITY-COMPATIBLE MODELING

As discussed in Section
2.1, the probabilistic descriptions of marginal distributions, spatial variability, and inter-parameter dependence are generally incompatible for multivariate non-Gaussian random fields. Therefore, it is

necessary to characterize these three sources of uncertainty within a probability-compatible framework. In this subsection, a correlation-function-based probability-compatible modeling approach is adopted.

To resolve the incompatibility between the marginal distributions and the spatial variability of non-Gaussian random fields, the *transformed correlation function* (Liu et al., 2026) is employed. This function represents the correlation structure of an underlying standard Gaussian field that can be directly transformed into the target non-Gaussian field while exactly preserving the prescribed marginal distribution and spatial correlation, without requiring any iterative procedure. The relationship between the target correlation function and the transformed correlation function is expressed as (Liu et al., 2026)

$$\rho = \frac{1}{\sigma^2}\left[\iint_0^1 F^{-1}(u_1)F^{-1}(u_2)c_{\mathrm{G}}\left(u_1,u_2;\tilde{\rho}\right)\mathrm{d}u_1\mathrm{d}u_2 - \mu^2\right]. \tag{5}$$

where $F(\cdot)$ denotes the marginal distribution of the target non-Gaussian field, $c_{\mathrm{G}}(\cdot)$ is the Gaussian copula density function, $\tilde{\rho}$ represents the transformed correlation function, and $\rho$ is the target correlation function of the non-Gaussian field. Equation (5) can be solved numerically to establish the relationship between $\rho(\boldsymbol{\xi})$ and $\tilde{\rho}(\boldsymbol{\xi})$, $\boldsymbol{\xi} = (\xi_1,\xi_2,\xi_3)^{\mathrm{T}}$ is the spatial lag vector.

Once the transformed correlation function $\tilde{\rho}(\boldsymbol{\xi})$ is obtained, it can be employed to generate an underlying standard Gaussian random field using any suitable random field simulation technique. Subsequently, the generated Gaussian field can be directly transformed into the target non-Gaussian field while preserving the prescribed marginal distribution and spatial correlation structure. This transformation forms the basis for the probability-compatible simulation of the concrete constitutive parameter fields considered in this study.

The probabilistic dependence of multivariate non-Gaussian random fields should also be characterized in a probability-compatible manner. Since the dependence structure among different variables is described using copula functions, the incompatibility between prescribed marginal distributions and copula-based dependence models must be properly addressed (Lyu et al., 2025a; Liu et al., 2026). To this end, a target non-Gaussian field can be constructed from a main dependent field and an independent standard Gaussian field through the bridge-function framework. Specifically,

$$Y_2(\boldsymbol{x}) = F_2^{-1}\left(C_{2|1}^{-1}\{\Phi[Z_{\mathrm{b}}(\boldsymbol{x})]|F_1[Y_1(\boldsymbol{x})]\}\right), \tag{6}$$

where $C_{2|1}^{-1}(\cdot)$ denotes the inverse conditional copula function, $Z_{\mathrm{b}}(\boldsymbol{x})$ represents the standard Gaussian bridge field with correlation function $\rho_{\mathrm{b}}(\boldsymbol{\xi})$, $Y_1(\boldsymbol{x})$ is the main non-Gaussian field, and $Y_2(\boldsymbol{x})$ denotes the dependent non-Gaussian field. According to Eq. (6), the dependent field is generated from the main field and the Gaussian bridge field. Consequently, a relationship exists among the correlation functions of the main field, the dependent field, and the Gaussian bridge field. To characterize this relationship, Lyu et al. (2025b) introduced the *bridge function* for multivariate Gaussian random fields, which was subsequently generalized to dependent non-Gaussian random fields by Liu et al. (2026). The resulting bridge function can be expressed as

$$\begin{aligned}\tilde{\rho}_2 &= r_{\mathrm{b}}\left(\rho_{\mathrm{b}}|\tilde{\rho}_1\right)\\ &= \iiiint_{[0,1]^4}\Phi^{-1}[C_{2|1}^{-1}(u_{\mathrm{b}}|u_1)]\Phi^{-1}[C_{2|1}^{-1}(u'_{\mathrm{b}}|u'_1)]c_{\mathrm{G}}(u_{\mathrm{b}},u'_{\mathrm{b}};\rho_{\mathrm{b}})c_{\mathrm{G}}\left(u_1,u'_1;\tilde{\rho}_1\right)\mathrm{d}u_1\mathrm{d}u_{\mathrm{b}}\mathrm{d}u'_1\mathrm{d}u'_{\mathrm{b}}.\end{aligned} \tag{7}$$

## 3. Numerically simulating three-dimensional random fields of concrete constitutive parameters

Uncertainty simulation is the indispensable computational counterpart to uncertainty modeling. Once the uncertainty modeling is finished, the uncertainty simulation can be carried out. This section presents the simulation procedure for the three-dimensional random fields of concrete constitutive parameters using the parsimonious stochastic harmonic function.

### 3.1. Parsimonious Stochastic Harmonic Function Representation

Conventional random field simulation methods, such as the spectral representation method (Shinozuka and Deodatis, 1996) and the Karhunen–Loève expansion (Phoon et al., 2005; Zheng et al., 2023), generally require a large number of basic random variables and considerable storage resources to accurately reproduce the target stochastic characteristics. This limitation becomes particularly severe for three-dimensional random fields, leading to high computational costs and reduced efficiency. To address this challenge, a parsimonious stochastic harmonic function (SHF) representation is employed in this study. The proposed approach incorporates Voronoi wavenumber partitioning (Song et al., 2019), spectrum-dependent random wavenumbers (Chen et al., 2020), and the concept of phase-evolution starting points (Li et al., 2013) into the SHF framework, thereby enabling an accurate and computationally efficient representation of three-dimensional random fields with only a small number of basic random variables. The parsimonious SHF representation is expressed as

$$\widetilde{Z}_{\mathrm{Voro}}^{\mathrm{SHF}}(\boldsymbol{x}) = \sum_{q=1}^{N} \sqrt{4S(\tilde{\boldsymbol{\kappa}}_q)\mathrm{Vol}(\Omega_q)} \sum_{m=1}^{4} \cos\left[(\boldsymbol{s}_m \odot \tilde{\boldsymbol{\kappa}}_q)^{\mathrm{T}} \boldsymbol{x} + \widetilde{\phi}_q^{(m)}\right], \tag{8}$$

where $\mathrm{Vol}(\Omega_q)$ denotes the volume of the Voronoi cell $\Omega_q$, $\boldsymbol{s}_1 = [1, 1, 1]^{\mathrm{T}}$, $\boldsymbol{s}_2 = [1, 1, -1]^{\mathrm{T}}$, $\boldsymbol{s}_3 = [1, -1, 1]^{\mathrm{T}}$, $\boldsymbol{s}_4 = [1, -1, -1]^{\mathrm{T}}$ are sign vectors used to generate different directional combinations of the wavenumbers, and the operator $\odot$ denotes the Hadamard (element-wise) product (Horn and Johnson, 2012); $\tilde{\boldsymbol{\kappa}}_q = (\tilde{\kappa}_{q,1}, \tilde{\kappa}_{q,2}, \tilde{\kappa}_{q,3})^{\mathrm{T}}$ are the spectrum-dependent wavenumbers generated by

$$\begin{cases} \tilde{\kappa}_{q,1} = (\kappa_{q,1}^{\mathrm{U}} - \kappa_{q,1}^{\mathrm{L}})\vartheta_1 \\ \tilde{\kappa}_{q,2} = (\kappa_{q,2}^{\mathrm{U}} - \kappa_{q,2}^{\mathrm{L}})\vartheta_2 \\ \tilde{\kappa}_{q,3} = (\kappa_{q,3}^{\mathrm{U}} - \kappa_{q,3}^{\mathrm{L}})\vartheta_3 \end{cases}, \tag{9}$$

where $\vartheta_1$, $\vartheta_2$, and $\vartheta_3$ are three independent random variables uniformly distributed over $[0, 1]$ (Chen et al., 2020); $\phi_q^{(m)}$ are the random phase angles generated by

$$\widetilde{\phi}_q^{(m)} = \mathrm{mod}\left[|\tilde{\boldsymbol{\kappa}}_q| \sqrt{4S(\tilde{\boldsymbol{\kappa}}_q)\mathrm{Vol}(\Omega_q)}\, \varpi_m, 2\pi\right], \tag{10}$$

where $\varpi_m$, $m$=1,2,3,4, are four independent $\mathit{\Gamma}$- distributed random variables with shape parameter $k_{\varpi} = 1.1$ and scale parameter $s_{\varpi} = 8.2 \times 10^8$ (Li et al., 2013). Together with the three spectrum-dependent random wavenumbers, the proposed parsimonious SHF representation requires only seven basic random variables to simulate a three-dimensional random field.

### 3.2. Parsimonious Simulation of Three-Dimensional Multivariate Constitutive Parameter Fields

First, the three dependent random fields, $f_{\mathrm{c}}(\boldsymbol{x})$, $\alpha_{\mathrm{c}}(\boldsymbol{x})$, and $\varepsilon_{\mathrm{c}}(\boldsymbol{x})$, are generated. The simulation procedure starts with the specification of the target marginal distributions, the transformed correlation functions of $f_{\mathrm{c}}(\boldsymbol{x})$, $\alpha_{\mathrm{c}}(\boldsymbol{x})$, and $\varepsilon_{\mathrm{c}}(\boldsymbol{x})$, as well as the vine-copula dependence structure among these constitutive parameters. Subsequently, the correlation functions of the Gaussian bridge fields $Z_{\mathrm{b}1}(\boldsymbol{x})$ and $Z_{\mathrm{b}3}(\boldsymbol{x})$ are determined using the bridge-function framework. Based on the obtained correlation functions, the bridge fields $Z_{\mathrm{b}1}(\boldsymbol{x})$, $Z_{\mathrm{b}3}(\boldsymbol{x})$, and the underlying Gaussian field $Z_{\alpha_{\mathrm{c}}}(\boldsymbol{x})$ are efficiently generated using the parsimonious SHF representation.

The generated Gaussian fields are then transformed into the target non-Gaussian constitutive parameter fields through the probability-compatible transformation framework described in Section 2. As a result, the simulated fields simultaneously preserve the prescribed marginal distributions, spatial correlation structures, and vine-copula-based dependence characteristics. The field $\alpha_{\mathrm{c}}(\boldsymbol{x})$ is first obtained through the marginal transformation of $Z_{\alpha_{\mathrm{c}}}(\boldsymbol{x})$. Subsequently, the prescribed dependence structure is incorporated sequentially according to the vine-copula hierarchy. The bridge field $Z_{\mathrm{b}2}(\boldsymbol{x})$ is generated conditionally on $Z_{\mathrm{b}1}(\boldsymbol{x})$ and $Z_{\mathrm{b}3}(\boldsymbol{x})$. Based on $\alpha_{\mathrm{c}}(\boldsymbol{x})$ and $Z_{\mathrm{b}1}(\boldsymbol{x})$, the compressive strength field $f_{\mathrm{c}}(\boldsymbol{x})$ is obtained through the inverse pair-copula transformation. Finally, the strain field $\varepsilon_{\mathrm{c}}(\boldsymbol{x})$ is constructed from $\alpha_{\mathrm{c}}(\boldsymbol{x})$ and $Z_{\mathrm{b}2}(\boldsymbol{x})$ using the inverse conditional-copula transformation.

Through this recursive construction procedure, the resulting fields $\alpha_{\mathrm{c}}(\boldsymbol{x})$, $f_{\mathrm{c}}(\boldsymbol{x})$, and $\varepsilon_{\mathrm{c}}(\boldsymbol{x})$ simultaneously satisfy the prescribed marginal distributions, spatial correlation structures, and nonlinear probabilistic dependence relationships. Owing to the parsimonious SHF representation, only seven basic random variables are required for each three-dimensional random field. Consequently, the entire trivariate constitutive-parameter model can be represented using only 21 basic random variables, including those associated with the bridge fields. Representative points in the resulting stochastic space are generated using the generalized F-discrepancy method. The overall simulation procedure is illustrated in Figure 1.

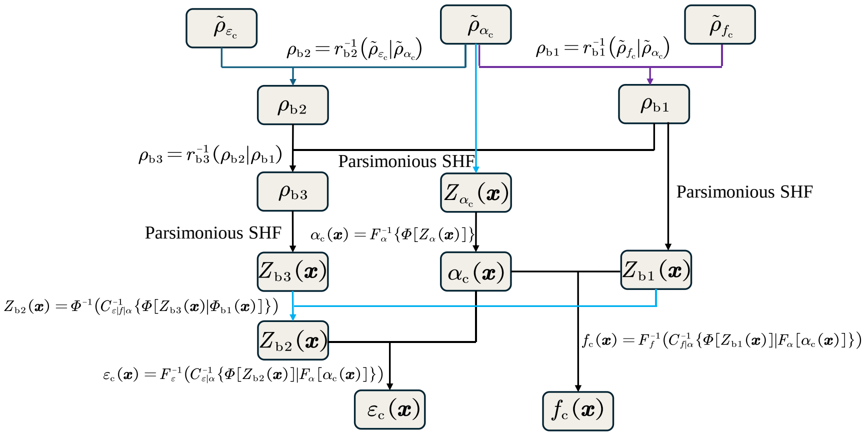
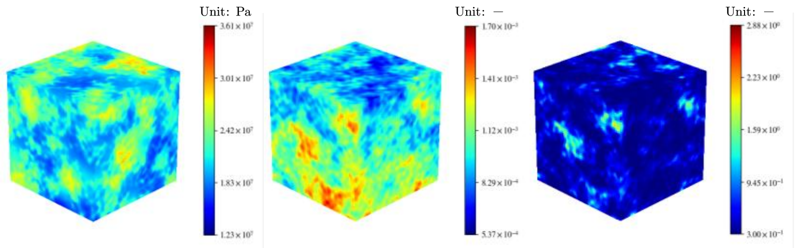


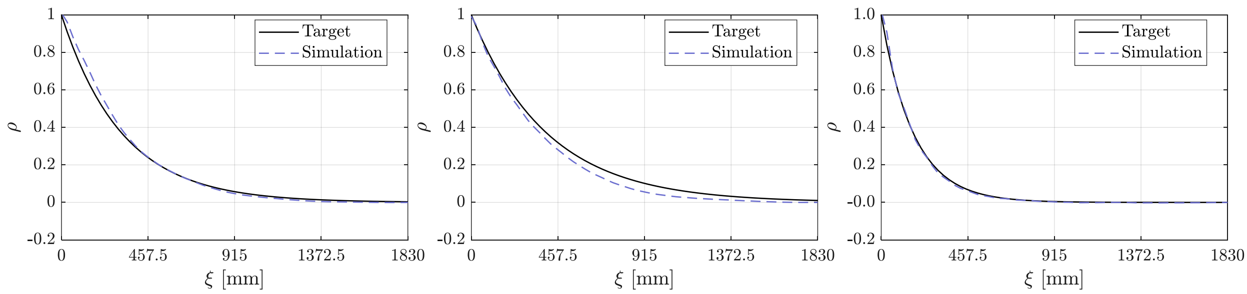

*Figure 1*. Simulation procedure for the dependent concrete constitutive parameter fields $\alpha_{\mathrm{c}}(\boldsymbol{x})$, $f_{\mathrm{c}}(\boldsymbol{x})$, and $\varepsilon(\boldsymbol{x})$.

Figure 2 presents representative realizations of the generated trivariate random fields, showing realistic spatial variability for $f_{\mathrm{c}}(\boldsymbol{x})$, $\varepsilon_{\mathrm{c}}(\boldsymbol{x})$, and $\alpha_{\mathrm{c}}(\boldsymbol{x})$. As shown in Figure 3, the simulated correlation functions agree closely with the target correlation functions, demonstrating that the prescribed spatial correlation structures are accurately reproduced. The comparisons of the target and simulated marginal probability density functions in Figure 4 indicate excellent agreement for all three variables. Furthermore, Figure 5 shows that the simulated samples closely follow the target joint probability distributions, confirming the ability of the proposed vine-copula-based framework to reproduce the prescribed nonlinear dependence structure. Overall, the results verify that the proposed method simultaneously preserves marginal distributions, spatial correlations, and probabilistic dependence while maintaining a highly parsimonious stochastic representation.



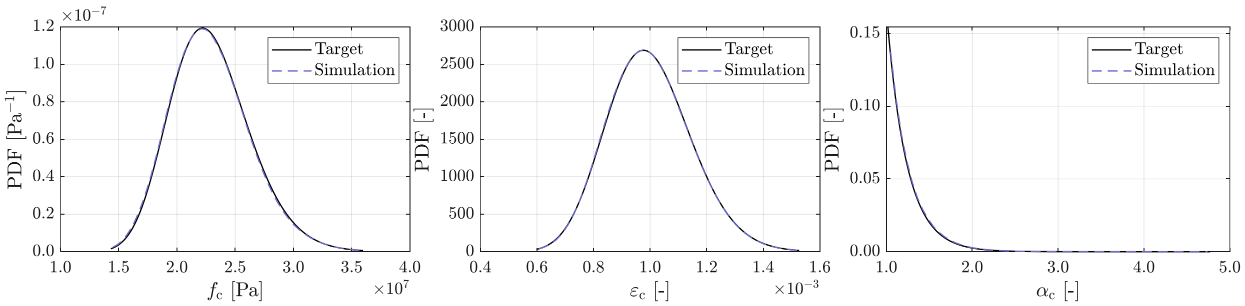

(a) Realization of $f_c(\boldsymbol{x})$. (b) Realization of $\varepsilon_c(\boldsymbol{x})$. (c) Realization of $\alpha_c(\boldsymbol{x})$.

*Figure 2*. Representative realizations of the generated trivariate random fields $\alpha_c(\boldsymbol{x})$, $f_c(\boldsymbol{x})$, and $\varepsilon(\boldsymbol{x})$.



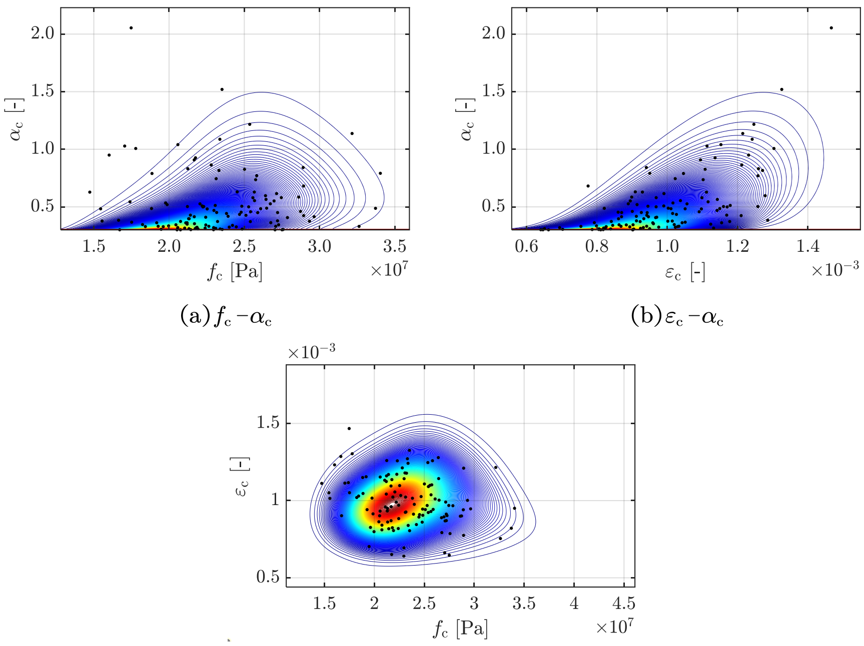

(a) Correlation function of $f_c(\boldsymbol{x})$. (b) Correlation function of $\varepsilon_c(\boldsymbol{x})$. (c) Correlation function of $\alpha_c(\boldsymbol{x})$.

*Figure 3*. Comparison of the target and simulated correlation functions.




(a) Marginal PDF of $f_c(\boldsymbol{x})$. (b) Marginal PDF of $\varepsilon_c(\boldsymbol{x})$. (c) Marginal PDF of $\alpha_c(\boldsymbol{x})$.

*Figure 4*. Comparison of the target marginal PDFs and the simulated results.

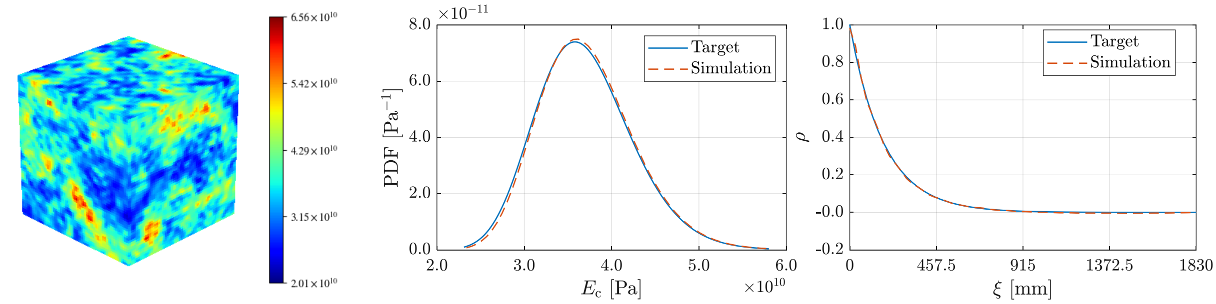



(a) $f_c - \alpha_c$ (b) $\varepsilon_c - \alpha_c$

(c) $\varepsilon_c - f_c$

*Figure 5*. Generated sample points obtained by the proposed method together with the target joint PDF contours.

The independent field $E(\boldsymbol{x})$ can be generated directly from its transformed correlation function. However, to ensure consistency with Drucker's postulate (Wan et al., 2020), the realizations of $E(\boldsymbol{x})$ are adjusted accordingly. The resulting realizations, together with comparisons between the target and simulated correlation functions and marginal probability density functions, are presented in Figure 6.




(a) Realization of $E_c(\boldsymbol{x})$. (b) Marginal PDF of $E_c(\boldsymbol{x})$. (c) Correlation function of $E_c(\boldsymbol{x})$.

*Figure 6*. Representative realization of $E(\boldsymbol{x})$ and comparisons of the marginal PDF and correlation function between target and simulated results.

## 4. Concluding remarks

In this paper, a probability-compatible and computationally efficient framework is proposed for the simulation of three-dimensional multivariate non-Gaussian random fields. The framework integrates the transformed correlation function, the bridge function, the vine-copula-based dependence model, and the parsimonious SHF representation into a unified modeling and simulation procedure. As a result, the generated random fields simultaneously and accurately reproduce the prescribed marginal distributions, spatial correlation structures, and nonlinear probabilistic dependence relationships.

Numerical investigations demonstrate that the proposed approach accurately reproduces the prescribed statistical characteristics, including the target marginal distributions, spatial correlation structures, and probabilistic dependence relationships. Furthermore, owing to the parsimonious SHF representation, only seven basic random variables are required for each three-dimensional random field, resulting in a substantial reduction in stochastic dimensionality and computational cost compared with conventional simulation techniques. Consequently, the four concrete constitutive parameter fields considered in this study can be represented using only 28 basic random variables in total, providing an efficient framework for large-scale uncertainty quantification and stochastic finite element analysis.

The proposed framework therefore provides a rigorous and efficient tool for the modeling and simulation of multivariate heterogeneous random media. Its ability to preserve complex probabilistic characteristics while maintaining high computational efficiency makes it particularly attractive for uncertainty quantification, reliability analysis, and stochastic finite element modeling of engineering materials and structures.

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