Efficient Probabilistic Response Analysis of Large-Scale Engineering Structures under Stochastic Excitation via the DR-PDEE Incorporating an Auxiliary Diffusion Process

Tingting Sun[1)] and Jianbing Chen[2)]

[1)] *State key laboratory of disaster reduction in Civil Engineering and College of Civil Engineering, Tongji University, Shanghai 200092, China; suntingting@tongji.edu.cn;*

[2)] *State key laboratory of disaster reduction in Civil Engineering and College of Civil Engineering, Tongji University, Shanghai 200092, China; chenjb@tongji.edu.cn*

Abstract. Probabilistic response analysis of engineering structures under stochastic excitation remains a critical and long-standing challenge. The main difficulties arise from the high dimensionality of structural systems, nonlinear material behavior, and uncertainties in both system parameters and external excitations. To address these challenges, an efficient framework is developed based on the dimension-reduced probability density evolution equation (DR-PDEE) incorporating an auxiliary diffusion process. The DR-PDEE is a low-dimensional partial differential equation (PDE) governing the instantaneous probability density function (PDF) of a path-continuous response of interest in a high-dimensional dynamical system. However, under non-white stochastic excitation, the intrinsic diffusion function of the response generally vanishes. This zero-diffusion issue complicates the numerical solution of the DR-PDEE. To overcome this difficulty, white noise statistically correlated with the original non-white excitation is constructed and applied to a simple stochastic system. The system output is then selected as the auxiliary diffusion process. Consequently, a two-dimensional DR-PDEE with nonzero diffusion is formulated for the joint process of the response of interest and the auxiliary variable. This formulation avoids the construction of an equivalent linear filter and provides high flexibility for complex non-white excitations. The proposed method is applied to a high-rise reinforced concrete (RC) frame-core tube structure subjected to stochastic near-field seismic excitation. Nonlinear constitutive behavior of both reinforcing steel and concrete is considered, together with uncertainties in the concrete material properties. The proposed method accurately captures the pronounced nonstationary and non-Gaussian features of the response PDFs. The results demonstrate its applicability and efficiency for probabilistic response analysis of large-scale nonlinear structures under complex non-white stochastic excitations.

Keywords: Nonlinear engineering structure, Non-white excitation, Stochastic response analysis, DR-PDEE, Auxiliary diffusion process.

1. Introduction

Engineering structures are frequently subjected to environmental dynamic actions with inherent randomness. In particular, these dynamic actions, such as seismic ground motions, turbulent winds, and ocean waves, often exhibit pronounced non-white and non-stationary characteristics (Rezaeian and Der Kiureghian, 2008; Li and Chen, 2009). Meanwhile, uncertainties in structural properties, such as material parameters, may significantly affect structural responses (Schuëller, 2007). Probabilistic response analysis is therefore essential for quantifying structural responses under stochastic excitation.

Despite extensive efforts, probabilistic response analysis of high-dimensional nonlinear engineering systems remains highly challenging. Simulation-based methods, such as Monte Carlo simulation (MCS), require numerous deterministic analyses and may become computationally prohibitive (Li and Wang, 2023). Even with advanced sampling strategies and surrogate models, these methods remain costly for large-scale engineering structures, as a single nonlinear time-history analysis may require substantial computational resources and be highly time-consuming (Moustapha et al., 2022). Another class of methods directly solves equations governing the evolution of response probability density functions (PDFs), such as the Fokker–Planck–Kolmogorov (FPK) equation (Gardiner, 1985). However, the classical FPK equation generally applies to Markov systems driven by white-noise excitation and typically becomes high-dimensional for engineering structures (Gardiner, 1985). Similar dimensionality issues arise in other classical PDF evolution equations, such as the Liouville equation (Soong, 1973) and the Dostupov–Pugachev equation (Dostupov and Pugachev, 1957). Analytical solutions to these equations are rarely available (Zhu et al., 1990), while their direct numerical solution is generally impractical for large-scale systems. Approximate methods, including stochastic linearization (Roberts and Spanos, 1990) and moment-closure techniques (Zhang et al., 2020), may lose accuracy for strongly nonlinear systems. The dimensionality problem therefore remains a central obstacle in stochastic analysis of nonlinear engineering structures.

Further, the non-white and non-stationary nature of stochastic excitations makes the stochastic response analysis more challenging. For example, applying the FPK equation to systems under non-white excitation generally requires filters to represent the excitation, which augment the system state and increase the equation dimensionality. Stochastic averaging can also be applied to certain non-white excitation problems, but it generally relies on restrictive assumptions regarding system damping and excitation bandwidth (Roberts and Spanos, 1986).

Moreover, simultaneous consideration of stochastic excitation and uncertain system parameters complicates the analysis. Such problems are commonly reformulated as stochastic structural analyses. One approach is to represent the stochastic excitation using a finite set of random variables. Alternatively, the conditional random vibration problem is first solved for fixed system parameters, and the parameter uncertainty is then propagated through the resulting conditional solutions. For high-dimensional nonlinear systems, however, both approaches generally lead to higher-dimensional or more complex stochastic analyses.

In this regard, the dimension-reduced probability density evolution equation (DR-PDEE) provides an efficient alternative for stochastic response analysis of high-dimensional nonlinear systems (Lyu and Chen, 2022). It governs the PDF of general path-continuous response processes and does not require these processes to be Markovian (Lyu and Chen, 2022; Luo et al., 2022b). Therefore, a low-dimensional DR-PDEE can be established for the path-continuous response process of interest, irrespective of the dimension of the original structural system and whether the system parameters are deterministic or random (Chen and Lyu, 2022).

The DR-PDEE also applies to structural responses under non-white excitation. However, when the non-white excitation has no singular component in its autocorrelation function, the intrinsic diffusion coefficient of the structural response vanishes. This results in a Liouville-type equation and complicates the numerical solution. Previous studies addressed this issue by representing the non-white excitation using a linear filter driven by white noise (Lyu and Chen, 2022; Chen et al., 2024; Luo et al., 2022a). The two-dimensional DR-PDEE was then formulated for the joint process consisting of the response of interest and one filter output. However, constructing such filters can be difficult, particularly for complex excitations without rational power spectral density (PSD) functions or with fully non-stationary characteristics (Luo et al., 2022b; Chen et al., 2024).

To overcome this limitation, this study introduces an auxiliary diffusion process method that avoids filter construction. The approach first constructs a white-noise process statistically correlated with the non-white excitation and uses it to drive a simple low-dimensional stochastic system. The resulting output serves as the auxiliary diffusion process. By combining the structural response of interest with this auxiliary process, a two-dimensional DR-PDEE can be established, and a non-zero diffusion term is retained in the equation. The response PDF can therefore be obtained without constructing an equivalent linear filter. To demonstrate the applicability of the proposed method to large-scale engineering structures, a reinforced concrete (RC) frame–core tube structure subjected to stochastic near-field pulse-type ground motions is investigated. Its probabilistic responses are obtained using the proposed method.

The remainder of this paper is organized as follows. The proposed DR-PDEE method incorporating an auxiliary diffusion process and its numerical procedure are introduced in Section 2. An application of the method to the stochastic response analysis of a high-rise RC structure under stochastic seismic excitation is presented in Section 3. Finally, concluding remarks are provided in Section 4.

2. DR-PDEE-based stochastic response analysis by incorporating an auxiliary diffusion process

2.1. Problem statement

Generally, engineering structures subjected to stochastic excitation can be modeled as multi-degree-of-freedom (MDOF) systems after spatial discretization by the finite element method (FEM). Therefore, an n-degree-of-freedom (n-DOF) structural system is considered here. Its equation of motion reads

$$\boldsymbol{M}\left(\boldsymbol{\Theta}\right)\ddot{\boldsymbol{X}}\left(t\right)+\boldsymbol{C}\left(\boldsymbol{X}\left(t\right),\dot{\boldsymbol{X}}\left(t\right),t;\boldsymbol{\Theta}\right)\dot{\boldsymbol{X}}\left(t\right)+\boldsymbol{f}\left(\boldsymbol{X}\left(t\right),\dot{\boldsymbol{X}}\left(t\right),t;\boldsymbol{\Theta}\right)=\boldsymbol{L}\boldsymbol{\xi}\left(t\right), \tag{1}$$

where $\boldsymbol{X}(t)$, $\dot{\boldsymbol{X}}(t)$, and $\ddot{\boldsymbol{X}}(t) \in \mathbb{R}^n$ denote the displacement, velocity, and acceleration vectors, respectively. The matrix $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ denotes the mass matrix, while $\boldsymbol{C} \in \mathbb{R}^{n\times n}$ denotes the damping matrix. The vector $\boldsymbol{f} \in \mathbb{R}^n$ represents the nonlinear restoring force. The stochastic excitation is denoted by $\boldsymbol{\xi}(t) \in \mathbb{R}^r$, and $\boldsymbol{L} \in \mathbb{R}^{n\times r}$ is the corresponding load-location matrix. The random vector $\boldsymbol{\Theta} \in \mathbb{R}^s$ characterizes uncertainty in the structural parameters. Consequently, $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{f}$ may depend on $\boldsymbol{\Theta}$. In this study, $\boldsymbol{\xi}(t)$ is considered to be a non-white stochastic process.

Denote $\dot{\boldsymbol{X}}(t)$ by $\boldsymbol{V}(t)$. The state vector is introduced as $\boldsymbol{Z}(t) = \left(\boldsymbol{X}^\top(t), \boldsymbol{V}^\top(t)\right)^\top \in \mathbb{R}^m$, where $m = 2n$. For simplicity, the mass matrix $\boldsymbol{M}$ is assumed to be deterministic. Eq. (1) can then be rewritten in the state-space form as

$$\mathrm{d}\boldsymbol{Z}(t) = \boldsymbol{a}(\boldsymbol{Z}(t), t; \boldsymbol{\Theta})\,\mathrm{d}t + \boldsymbol{B}\boldsymbol{\xi}(t)\,\mathrm{d}t, \tag{2}$$

where

$$\boldsymbol{a}(\boldsymbol{Z}(t), t; \boldsymbol{\Theta}) = \begin{pmatrix} \boldsymbol{V}(t) \\ -\boldsymbol{M}^{-1}\left[\boldsymbol{C}(\boldsymbol{X}(t), \boldsymbol{V}(t), t; \boldsymbol{\Theta})\boldsymbol{V}(t) + \boldsymbol{f}(\boldsymbol{X}(t), \boldsymbol{V}(t), t; \boldsymbol{\Theta})\right] \end{pmatrix}, \tag{3}$$

and

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{0}_{n\times r} \\ \boldsymbol{M}^{-1}\boldsymbol{L} \end{bmatrix}. \tag{4}$$

Here, $\boldsymbol{0}n \times r$ denotes the $n \times r$ zero matrix. Only additive excitation is considered in this study; namely, $\boldsymbol{L}$ is independent of $\boldsymbol{X}(t)$ and $\dot{\boldsymbol{X}}(t)$. Therefore, different interpretations of stochastic integration with respect to $\boldsymbol{\xi}(t)$ need not be distinguished (Gardiner, 1985).

Accurate assessment of structural safety requires reliable estimation of the instantaneous PDFs of system responses. This is the main objective of this study.

2.2. DR-PDEE for the response of interest and the zero-diffusion issue

Determining the probabilistic response of the nonlinear system governed by Eq. (2) is challenging, particularly when the instantaneous PDF of a response of interest is required. The difficulty becomes especially severe for systems with a large number of DOFs n or a high-dimensional state space of dimension m. Nevertheless, the DR-PDEE provides an efficient tool for addressing this problem. In this subsection, the DR-PDEE for a general path-continuous process is first introduced, followed by its formulation for a response component of the state vector of Eq. (2). The zero-diffusion issue arising from non-white excitation is then discussed.

2.2.1. *DR-PDEE for general path-continuous processes*

First, consider a one-dimensional stochastic process, denoted by $Y(t) \in \mathbb{R}$. Assume that $Y(t)$ satisfies the Dynkin–Kinney condition (Dynkin, 1952; Kinney, 1953); namely, for any $\varepsilon > 0$,

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{|z-y|>\varepsilon} p_Y(z, t + \Delta t \mid y, t)\,\mathrm{d}z = 0 \tag{5}$$

holds uniformly for all y and t. The Dynkin–Kinney condition, also known as the path-continuity condition (Seregin, 1961), implies that the sample paths of $Y(t)$ are continuous in t with probability one.

According to Ref. (Lyu and Chen, 2022), the instantaneous PDF of $Y(t)$ satisfying the Dynkin–Kinney condition is governed by

$$\frac{\partial p_Y(y,t)}{\partial t} = -\frac{\partial a_Y^{\mathrm{int}}(y,t)\,p_Y(y,t)}{\partial y} + \frac{1}{2}\frac{\partial^2 b_Y^{\mathrm{int}}(y,t)\,p_Y(y,t)}{\partial y^2}. \tag{6}$$

Eq. (6) is referred to as the dimension-reduced probability density evolution equation (DR-PDEE) for the path-continuous stochastic process $Y(t)$. Here, $a_Y^{\text{int}}(y,t)$ and $b_Y^{\text{int}}(y,t)$ denote the intrinsic drift and diffusion functions of $Y(t)$, respectively. They are defined as the first- and second-order conditional derivate moments of $Y(t)$ (Lyu and Chen, 2022; Gardiner, 1985):

$$\begin{cases} a_Y^{\text{int}}(y,t) = \lim\limits_{\Delta t \to 0} \dfrac{1}{\Delta t} \mathbb{E}\left(Y(t+\Delta t) - Y(t) \middle| Y(t) = y \right), \\ b_Y^{\text{int}}(y,t) = \lim\limits_{\Delta t \to 0} \dfrac{1}{\Delta t} \mathbb{E}\left\{ \left[Y(t+\Delta t) - Y(t) \right]^2 \middle| Y(t) = y \right\}. \end{cases} \tag{7}$$

In Eq. (7), $\mathbb{E}$ denotes mathematical expectation.

The DR-PDEE for the multidimensional case takes a similar form. Consider a d-dimensional stochastic process $\boldsymbol{Y}(t) \in \mathbb{R}^d$. Assume that $\boldsymbol{Y}$ also satisfies the Dynkin–Kinney condition (Dynkin, 1952; Kinney, 1953). Specifically, for any $\varepsilon > 0$,

$$\lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_{\|\boldsymbol{z}-\boldsymbol{y}\| > \varepsilon} p_{\boldsymbol{Y}}(\boldsymbol{z}, t+\Delta t \mid \boldsymbol{y}, t) \, \mathrm{d}\boldsymbol{z} = 0 \tag{8}$$

holds uniformly for all $\boldsymbol{y}$ and t, where $\|\cdot\|$ denotes the Euclidean norm. Under this condition, the instantaneous PDF of $\boldsymbol{Y}(t)$, denoted by $p_{\boldsymbol{Y}}(\boldsymbol{y},t)$, is governed by the following DR-PDEE (Lyu and Chen, 2022):

$$\frac{\partial p_{\boldsymbol{Y}}(\boldsymbol{y},t)}{\partial t} = -\sum_{i=1}^{d} \frac{\partial a_{\boldsymbol{Y},i}^{\text{int}}(\boldsymbol{y},t) \, p_{\boldsymbol{Y}}(\boldsymbol{y},t)}{\partial y_i} + \frac{1}{2} \sum_{i=1}^{d} \sum_{j=1}^{d} \frac{\partial^2 b_{\boldsymbol{Y},ij}^{\text{int}}(\boldsymbol{y},t) \, p_{\boldsymbol{Y}}(\boldsymbol{y},t)}{\partial y_i \partial y_j}, \tag{9}$$

where

$$\begin{cases} a_{\boldsymbol{Y},i}^{\text{int}}(\boldsymbol{y},t) = \lim\limits_{\Delta t \to 0} \dfrac{1}{\Delta t} \mathbb{E}\left(Y_i(t+\Delta t) - Y_i(t) \middle| \boldsymbol{Y}(t) = \boldsymbol{y} \right), \\ b_{\boldsymbol{Y},ij}^{\text{int}}(\boldsymbol{y},t) = \lim\limits_{\Delta t \to 0} \dfrac{1}{\Delta t} \mathbb{E}\left\{ \left[Y_i(t+\Delta t) - Y_i(t) \right] \left[Y_j(t+\Delta t) - Y_j(t) \right] \middle| \boldsymbol{Y}(t) = \boldsymbol{y} \right\}. \end{cases} \tag{10}$$

Note that the DR-PDEEs in Eqs. (6) and (9) do not require $Y(t)$ and $\boldsymbol{Y}(t)$, respectively, to be Markovian (Lyu and Chen, 2022; Chen and Lyu, 2022; Luo et al., 2022b).

2.2.2. *DR-PDEE for path-continuous response processes under non-white excitation*

Next, consider the state response process governed by Eq. (2). Since the DR-PDEE applies to general path-continuous processes regardless of whether they are Markovian, a low-dimensional DR-PDEE can be established for any response component of interest.

For example, let the ℓ-th component of $\boldsymbol{Z}(t)$ in Eq. (2) be the response of interest, denoted by $Z_\ell(t)$. By setting $Y(t) = Z_\ell(t)$, the corresponding one-dimensional DR-PDEE governing its PDF $p_{Z_\ell}(z_\ell, t)$ takes the form of Eq. (6). The intrinsic drift and diffusion functions of $Z_\ell(t)$ are examined below. According to Eq. (2), the increment of $Z_\ell(t)$ over a small time interval Δt is expressed as

$$\Delta Z_\ell(t) = Z_\ell(t+\Delta t) - Z_\ell(t) = \int_t^{t+\Delta t} a_\ell(\boldsymbol{Z}(\tau), \tau; \boldsymbol{\Theta}) \, \mathrm{d}\tau + \int_t^{t+\Delta t} \left[\boldsymbol{B}\boldsymbol{\xi}(\tau) \right]_\ell \mathrm{d}\tau. \tag{11}$$

Further, define the integral process of the non-white excitation $\boldsymbol{\xi}(t)$ as $\boldsymbol{\Xi}(t) = \int_0^t \boldsymbol{\xi}(\tau)\,\mathrm{d}\tau$. Assume that $\boldsymbol{\Xi}(t)$ is mean-square continuous. This assumption is satisfied when the autocorrelation function of $\boldsymbol{\xi}(t)$, denoted by $\boldsymbol{R}_{\boldsymbol{\xi}}(t_1, t_2) = \mathbb{E}\left(\boldsymbol{\xi}(t_1)\,\boldsymbol{\xi}^\top(t_2)\right)$, contains no Dirac-$\delta$-type singularity at $t_1 = t_2$, unlike that of white noise. This condition is generally satisfied by physical random excitations. For example, wind loads, ocean waves, and realistic earthquake ground motions are non-white processes with finite mean-square values, and their autocorrelation functions contain no such Dirac-δ-type singularity. Therefore, the integral processes of these excitations are generally mean-square continuous.

Under this condition, substituting Eq. (11) into the definitions of the intrinsic drift and diffusion functions yields (Lyu, 2022)

$$\begin{cases} a_{Z_\ell}^{\mathrm{int}}(z_\ell, t) = \lim\limits_{\Delta t \to 0} \dfrac{1}{\Delta t}\mathbb{E}\left(\Delta Z_\ell(t) \middle| Z_\ell(t) = z_\ell\right) = \mathbb{E}\left(a_\ell(\boldsymbol{Z}(t), t; \boldsymbol{\Theta}) + [\boldsymbol{B}\boldsymbol{\xi}(t)]_\ell \middle| Z_\ell(t) = z_\ell\right), \\ b_{Z_\ell}^{\mathrm{int}}(z_\ell, t) = \lim\limits_{\Delta t \to 0} \dfrac{1}{\Delta t}\mathbb{E}\left\{[\Delta Z_\ell(t)]^2 \middle| Z_\ell(t) = z_\ell\right\} = 0. \end{cases} \tag{12}$$

It follows that the intrinsic diffusion function degenerates to zero when $\left[\boldsymbol{B}\boldsymbol{R}_{\boldsymbol{\xi}}(t_1, t_2)\,\boldsymbol{B}^\top\right]_{\ell\ell}$ contains no Dirac-δ-type singularity at $t_1 = t_2$. In this case, the excitation-induced response increment is of order Δt, and its second-order conditional moment is therefore of order $(\Delta t)^2$. Dividing this moment by Δt and taking the limit as $\Delta t \to 0$ consequently gives zero. Accordingly, the DR-PDEE for $Z_\ell(t)$ reduces to the following Liouville-type PDE:

$$\frac{\partial p_{Z_\ell}(z_\ell, t)}{\partial t} = -\frac{\partial a_{Z_\ell}^{\mathrm{int}}(z_\ell, t)\, p_{Z_\ell}(z_\ell, t)}{\partial z_\ell}. \tag{13}$$

In principle, Eq. (13) can still be solved numerically. However, the tails of the resulting PDF are sensitive to the diffusion term (Lyu, 2022). When the intrinsic diffusion function vanishes, the numerical solution may be less robust than that of an equation with nonzero diffusion. This issue is particularly evident when the DR-PDEE is solved using the path integral solution (PIS) technique, because the zero diffusion term makes construction of the short-time transition PDF less straightforward (Yu et al., 1997). Similar difficulties may also arise in other numerical schemes. Therefore, a key objective of this study is to formulate a low-dimensional DR-PDEE with a nonzero intrinsic diffusion function. The auxiliary diffusion process introduced in the next subsection is constructed for this purpose.

2.3. The DR-PDEE for the state response of interest incorporating an auxiliary diffusion process

2.3.1. *Constructing white noise correlated with the non-white excitation*

The auxiliary diffusion process is introduced to retain a nonzero intrinsic diffusion function. In previous studies, a possible approach is to represent the non-white excitation $\boldsymbol{\xi}(t)$ using a linear filter driven by white noise. The filter parameters are selected such that its output reproduces the statistical properties of $\boldsymbol{\xi}(t)$ (Luo et al., 2022b; Chen et al., 2024). However, this approach becomes difficult to apply to excitations with irrational PSDs or fully non-stationary characteristics, both of which are commonly encountered in practical engineering problems.

A more flexible method is developed based on the spectral representation of the non-white excitation. The desired white noise is then constructed by modifying the spectral amplitudes while retaining the same random phases. For simplicity, a one-dimensional non-white excitation $\xi_{\mathrm{nw}}(t)$ is considered first. It can be represented using the classical spectral representation (Shinozuka, 1971) method as

$$\xi_{\mathrm{nw}}(t) \approx \sum_{i=1}^{N} A_i \cos(\omega_i t + \Phi_i), \tag{14}$$

where $\{\Phi_i\}_{i=1}^{N}$ are independent random phase angles uniformly distributed over $[0, 2\pi]$. For a stationary excitation, the spectral amplitude is given by (Shinozuka, 1971)

$$A_i(\omega) = \sqrt{\frac{2S_{\mathrm{nw}}(\omega_i)\,\Delta\omega_i}{\pi}}, \tag{15}$$

where $S_{\mathrm{nw}}(\omega)$ denotes the PSD of $\xi_{\mathrm{nw}}(t)$.

The key idea here is to retain the same random phase angles $\{\Phi_i\}_{i=1}^{N}$ while replacing the spectral amplitudes $A_i(\omega)$ or $A_i(\omega, t)$ with those corresponding to a constant white-noise spectrum. In this way, an approximate white noise process, denoted by $\xi_{\mathrm{w}}(t)$, can be constructed as

$$\xi_{\mathrm{w}}(t) \approx \sum_{i=1}^{N} \sqrt{\frac{2D_0\Delta\omega_i}{\pi}} \cos(\omega_i t + \Phi_i), \tag{16}$$

where D_0 denotes the intensity of the white noise $\xi_{\mathrm{w}}(t)$. Because Eqs. (14) and (16) share the same random phase angles, the constructed white noise is statistically correlated with the original non-white excitation.

The above spectral representations are discrete approximations. In a formal continuous form, the non-white excitation can be expressed through the white noise by

$$\xi_{\mathrm{nw}}(t) = \mathcal{F}^{-1}\left[\sqrt{\frac{S_{\mathrm{nw}}(\omega)}{D_0}}\mathcal{F}(\xi_{\mathrm{w}}(t))\right], \tag{17}$$

where $\mathcal{F}$ denotes the Fourier transform. In the classical sense, the Fourier transform is defined for absolutely integrable functions, whereas white noise does not possess a classical Fourier transform. Therefore, Eq. (17) should be interpreted in the generalized-function sense, under which the symbolic operation remains consistent with its classical counterpart.

Since $\xi_{\mathrm{nw}}(t)$ and $\xi_{\mathrm{w}}(t)$ share the same random phases, they are correlated. For stationary cases, their cross-correlation function is

$$\mathbb{E}[\xi_{\mathrm{nw}}(t_1)\,\xi_{\mathrm{w}}(t_2)] = \frac{1}{\pi}\int_0^{\infty} \sqrt{S_{\mathrm{nw}}(\omega)\,D_0}\cos(\omega(t_1 - t_2))\,\mathrm{d}\omega. \tag{18}$$

For a non-stationary excitation characterized by the evolutionary PSD $S_{\mathrm{nw}}(\omega, t)$, the above representation retains the same form, whereas the spectral amplitude becomes time-dependent as (Liang et al., 2007)

$$A_i(\omega, t) = \sqrt{\frac{2S_{\mathrm{nw}}(\omega_i, t)\,\Delta\omega_i}{\pi}}. \tag{19}$$

2.3.2. *Constructing the auxiliary diffusion process from the correlated white noise*
Next, the constructed white noise $\xi_{\mathrm{w}}(t)$ is taken as the input to a simple stochastic system, yielding the Itô stochastic differential equation (SDE) governing the auxiliary diffusion process:

$$\mathrm{d}\boldsymbol{U}(t) = -\boldsymbol{\alpha}(\boldsymbol{U}(t))\,\mathrm{d}t + \boldsymbol{\beta}\mathrm{d}W_0(t), \tag{20}$$

where $\boldsymbol{U}(t), \boldsymbol{\alpha}, \boldsymbol{\beta} \in \mathbb{R}^{n_{\boldsymbol{U}}}$, and $W_0(t) = \int_0^t \xi_{\mathrm{w}}(\tau)\,\mathrm{d}\tau$. As long as $\boldsymbol{\beta}$ is not identically zero, at least one component of $\boldsymbol{U}(t)$ is a diffusion process with a nonzero intrinsic diffusion function. This component can therefore be selected as the desired auxiliary diffusion process.

2.3.3. *DR-PDEE for the response of interest combined with the auxiliary diffusion process*
Further, the original state equation in Eq. (2) is combined with Eq. (20), together with the transformation between the white noise $\xi_{\mathrm{w}}(t)$ and non-white excitations $\xi_{\mathrm{nw}}(t)$. The resulting augmented system is

$$\begin{cases} \mathrm{d}\boldsymbol{Z}(t) = \boldsymbol{a}(\boldsymbol{Z}(t), t; \boldsymbol{\Theta})\,\mathrm{d}t + \boldsymbol{B}\xi_{\mathrm{nw}}(t)\,\mathrm{d}t, \\ \xi_{\mathrm{nw}}(t) = \mathcal{G}(\xi_{\mathrm{w}}(t)) \\ \mathrm{d}\boldsymbol{U}(t) = -\boldsymbol{\alpha}(\boldsymbol{U}(t))\,\mathrm{d}t + \boldsymbol{\beta}\mathrm{d}W_0(t). \end{cases} \tag{21}$$

In Eq. (21), the operator $\mathcal{G}$ represents the transformation in Eq. (17). Although the augmented system in Eq. (21) involves white noise, it is generally non-Markovian because the non-white excitation $\xi_{\mathrm{nw}}(t)$ depends on the history of $\xi_{\mathrm{w}}(t)$ through the transformation $\mathcal{G}$. It should also be noted that the state equations governing $\boldsymbol{Z}(t)$ and $\boldsymbol{U}(t)$ are dynamically decoupled. Nevertheless, the processes $\boldsymbol{Z}(t)$ and $\boldsymbol{U}(t)$ are statistically correlated because their respective excitations, $\xi_{\mathrm{nw}}(t)$ and $\xi_{\mathrm{w}}(t)$, are correlated.

Based on the above formulation, the response of interest $Z_\ell(t)$ is combined with one component of $\boldsymbol{U}(t)$, denoted by $U_\kappa(t)$, where $\kappa \in \{1, \ldots, n_{\boldsymbol{U}}\}$. The process $U_\kappa(t)$ is selected as the auxiliary diffusion process and must satisfy $\beta_\kappa \neq 0$. Here, β_κ is the κ-th component of $\boldsymbol{\beta}$.

By taking $\boldsymbol{Y}(t)$ in Eq. (9) as the joint process $(Z_\ell(t), U_\kappa(t))^\top$, the DR-PDEE governing its instantaneous joint PDF $p_{Z_\ell U_\kappa}(z_\ell, u_\kappa, t)$ is formulated as

$$\begin{aligned} \frac{\partial p_{Z_\ell U_\kappa}(z_\ell, u_\kappa, t)}{\partial t} = &-\frac{\partial a^{\mathrm{int}}_{Z_\ell U_\kappa,1} p_{Z_\ell U_\kappa}(z_\ell, u_\kappa, t)}{\partial z_\ell} - \frac{\partial a^{\mathrm{int}}_{Z_\ell U_\kappa,2} p_{Z_\ell U_\kappa}(z_\ell, u_\kappa, t)}{\partial u_\kappa} \\ &+\frac{1}{2}\frac{\partial^2 \left[\beta_\kappa^2 D_0\right] p_{Z_\ell U_\kappa}(z_\ell, u_\kappa, t)}{\partial {u_\kappa}^2}. \end{aligned} \tag{22}$$

In Eq. (22), the intrinsic drift functions are

$$\begin{cases} a^{\mathrm{int}}_{Z_\ell U_\kappa,1} = \mathbb{E}\left\{ a_\ell(\boldsymbol{Z}(t), t; \boldsymbol{\Theta}) + B_\ell \xi_{\mathrm{nw}}(t) \middle| Z_\ell(t) = z_\ell, U_\kappa(t) = u_\kappa \right\}, \\ a^{\mathrm{int}}_{Z_\ell U_\kappa,2} = \mathbb{E}\left\{ -\alpha_\kappa(\boldsymbol{U}(t)) \middle| Z_\ell(t) = z_\ell, U_\kappa(t) = u_\kappa \right\} \\ \qquad\qquad +\beta_\kappa \lim_{\Delta t \to 0} \frac{1}{\Delta t}\mathbb{E}\left\{ \int_t^{t+\Delta t} \xi_{\mathrm{w}}(\tau)\,\mathrm{d}\tau \middle| Z_\ell(t) = z_\ell, U_\kappa(t) = u_\kappa \right\}. \end{cases} \tag{23}$$

The diagonal intrinsic diffusion coefficients are $b^{\mathrm{int}}_{Z_\ell U_\kappa,11} = 0$ and $b^{\mathrm{int}}_{Z_\ell U_\kappa,22} = \beta_\kappa^2 D_0$. The cross intrinsic diffusion coefficient $b^{\mathrm{int}}_{Z_\ell U_\kappa,12}$ is given by

$$b^{\mathrm{int}}_{Z_\ell U_\kappa,12} = B_\ell \beta_\kappa \lim_{\Delta t \to 0} \frac{1}{\Delta t} \int_t^{t+\Delta t} \int_t^{t+\Delta t} \mathbb{E}\left\{ \xi_{\mathrm{nw}}(\tau_1)\,\xi_{\mathrm{w}}(\tau_2) \middle| Z_\ell(t) = z_\ell, U_\kappa(t) = u_\kappa \right\} \mathrm{d}\tau_1 \mathrm{d}\tau_2 \tag{24}$$

If the ℓ-th row of $\boldsymbol{B}$ is zero, i.e., $B_\ell = 0$, the cross intrinsic diffusion term becomes $b^{\text{int}}_{Z_\ell U_\kappa,12} = 0$. Otherwise, the cross term should be examined. A sufficient condition is

$$\int_t^{t+\Delta t} \int_t^{t+\Delta t} \mathbb{E}\left\{ \xi_{\text{nw}}(\tau_1)\, \xi_{\text{w}}(\tau_2) \middle| Z_\ell(t) = z_\ell, U_\kappa(t) = u_\kappa \right\} \mathrm{d}\tau_1 \mathrm{d}\tau_2 = o(\Delta t), \tag{25}$$

where $o(\Delta t)$ denotes a higher-order term than Δt. Under this condition, the cross intrinsic diffusion term also vanishes after division by Δt and taking the limit $\Delta t \to 0$.

Thus, a two-dimensional DR-PDEE is established for $Z_\ell(t)$ and $U_\kappa(t)$. The auxiliary process introduces the nonzero intrinsic diffusion coefficient $\beta_\kappa^2 D_0$ and prevents complete degeneration of the diffusion term. Solving Eq. (22) yields the joint PDF of $(Z_\ell(t), U_\kappa(t))^\top$. The PDF of $Z_\ell(t)$ is then obtained by

$$p_{Z_\ell}(z_\ell, t) = \int_{-\infty}^{\infty} p_{Z_\ell U_\kappa}(z_\ell, u_\kappa, t)\, \mathrm{d}u_\kappa. \tag{26}$$

2.3.4. *SDE governing the auxiliary diffusion process*

The above derivation allows some flexibility in selecting the SDE for the auxiliary diffusion process. In particular, the functions $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ in Eq. (20) can be chosen according to the following guidelines:

- Eq. (20) should be as simple as possible to limit the additional computational cost.
- The auxiliary process should facilitate accurate estimation of the intrinsic drift functions while retaining a nonzero intrinsic diffusion coefficient.

The second guideline can be understood from Eq. (23). The first component of the intrinsic drift function, $a^{\text{int}}_{Z_\ell U_\kappa,1}$, depends on the statistical dependence among $Z_\ell(t)$, $U_\kappa(t)$, and $a_\ell(\boldsymbol{Z}(t), t; \boldsymbol{\Theta}) + B_\ell \xi_{\text{nw}}(t)$, where the last quantity represents the instantaneous velocity of $Z_\ell(t)$. Therefore, the governing equation for $\boldsymbol{U}(t)$ should be selected such that $U_\kappa(t)$ is strongly correlated with this velocity. This correlation reduces the scatter of the samples used to estimate the conditional expectation. Consequently, $a^{\text{int}}_{Z_\ell U_\kappa,1}$ can be estimated more accurately with a limited number of deterministic analyses.

Following these guidelines, a simple choice is the one-dimensional Ornstein–Uhlenbeck-type Itô SDE (Sun et al., 2025)

$$\mathrm{d}U(t) = -\alpha_0 U(t)\, \mathrm{d}t + \beta \mathrm{d}W_0(t). \tag{27}$$

In this case, $U(t)$ itself serves as the auxiliary diffusion process. Another choice is a linear single-degree-of-freedom system governed by

$$\begin{cases} \mathrm{d}U_1(t) = U_2(t)\, \mathrm{d}t \\ \mathrm{d}U_2(t) = -\left[\omega_0^2 U_1(t) + 2\zeta_0 \omega_0 U_2(t)\right] \mathrm{d}t + \beta_2 \mathrm{d}W_0(t). \end{cases} \tag{28}$$

Here, $U_2(t)$ is selected as the auxiliary diffusion process.

To satisfy the second guideline, the parameters α_0, ω_0, and ζ_0 may be selected with reference to the characteristic time scale or dominant frequency of the original system in Eq. (2).

2.4. Numerical Procedure

The proposed method determines the PDF of the response of interest under non-white excitation through the following four steps:

1. Generate N_{smp} samples of $\mathbf{\Theta}$ and N_{smp} paired trajectories of $\xi_{\mathrm{nw}}(t)$ and $\xi_{\mathrm{w}}(t)$ using Eqs. (14) and (16). The same random phase angles are used for each pair.

2. For each sample, numerically integrate the augmented system in Eq. (21). Record the trajectories of $Z_{\ell}(t)$ and $U_{\kappa}(t)$, together with the quantities required to evaluate the intrinsic drift functions.

3. Estimate the intrinsic drift functions of $\left(Z_{\ell}(t), U_{\kappa}(t)\right)^{\top}$ in Eq. (23) from the N_{smp} response trajectories at each time step. Various estimators of conditional expectation can be used. In this study, locally weighted least-squares regression (LOWESS) is adopted (Cleveland and Devlin, 1988).

4. Substitute the estimated intrinsic drift functions into Eq. (22) and solve the two-dimensional DR-PDEE numerically. The PIS technique is used in this study (Yu et al., 1997). The PDF of $Z_{\ell}(t)$ is then obtained by evaluating the integral in Eq. (26).

Note that, when estimating the intrinsic drift functions in Eq. (23), the conditional expectations must first be evaluated prior to numerical differentiation for estimating the time derivatives. A detailed numerical scheme can be found in Ref. (Sun et al., 2025).

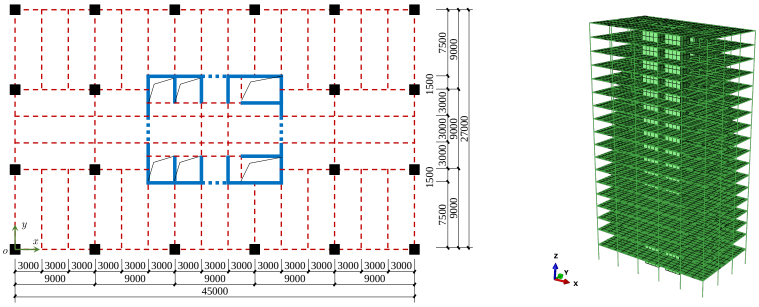
3. Engineering application: High-rise RC frame–core tube structure subjected to near-field seismic excitation

To demonstrate the proposed method in an engineering application, a high-rise RC frame–core tube structure subjected to stochastic near-field seismic excitation is considered. The uncertainties in both the material properties and seismic excitation are taken into account. The structural model is first introduced, followed by the probabilistic response using the proposed method.

3.1. Finite element model

The case study considers an 18-story high-rise RC frame–core tube structure. The total structural height is 84.0 m. The first two stories are 6.0 m high, while each of the remaining stories is 4.5 m high. The slab thickness is 120 mm. The structural plan layout is shown in Fig. (1a). The finite element (FE) model is shown in Fig. (1b). In this model, beams and columns are modeled using Timoshenko beam elements, whereas shear walls and slabs are modeled using layered shell elements. The FE model consists of 29312 elements, including 16856 beam elements and 12456 layered shell elements. It contains 13412 nodes and 80472 unconstrained DOFs.

Gravity loads, slab live loads, and curtain-wall loads are considered in the structural design. The slab live load is 3.5 kN/m^2, and the curtain-wall load is 3.5 kN/m along the perimeter of



(a) Structural plan layout. (b) FE model.

Figure 1. High-rise RC frame–core tube structure considered in the case study.

the structural plan. The stochastic seismic excitation is then applied to evaluate the probabilistic dynamic response of the structure.

3.2. Structural material properties and their probabilistic model

The structure consists of reinforcing steel and concrete. The reinforcing steel is treated as deterministic owing to its relatively low variability. It follows a bilinear elastic–plastic constitutive law, with its density, initial Young's modulus, yield strength, and ultimate strength taken as $7800\,\mathrm{kg/m^3}$, 205 GPa, 335 MPa, and 470 MPa, respectively.

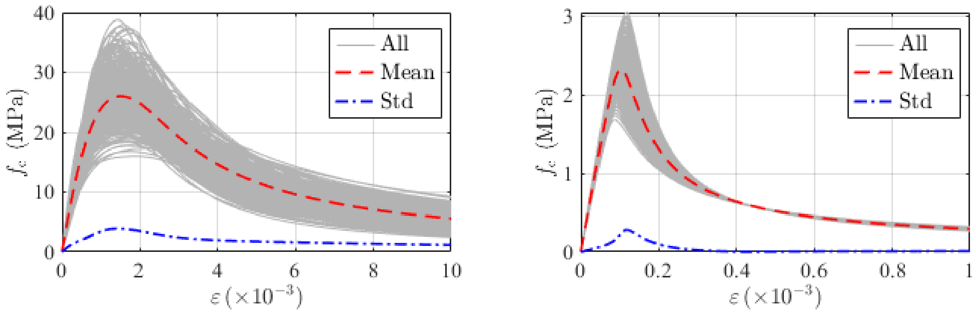
In contrast, concrete is considered as a random material because of the pronounced variability observed in its constitutive behavior (Li et al., 2014). In this structure, the concrete strength grade is C40 for walls and columns and C30 for beams and slabs. The constitutive behavior of concrete is described by the bi-scalar elastoplastic damage model (Li et al., 2014). This model characterizes the multiaxial nonlinear behavior of concrete and its parameters need to be calibrated using uniaxial compressive and tensile stress–strain relations.

In this study, the uniaxial compressive and tensile stress–strain relationships of concrete follow the constitutive model recommended by the Chinese Code for Design of Concrete Structures (GB 50010–2015) (Ministry of Housing and Urban-Rural Development of the People's Republic of China, 2015). The compressive stress–strain curve is governed by four parameters: the initial Young's modulus E_{c}, the compressive strength f_{cu}, the compressive peak strain $\varepsilon_{\mathrm{cu}}$, and the shape parameter governing its descending stage α_{c}. Similarly, the tensile stress–strain curve is governed by E_{c}, the tensile strength f_{tu}, the tensile peak strain $\varepsilon_{\mathrm{tu}}$, and the corresponding shape parameter for its descending regime α_{t}. Thus, the constitutive model involves seven material parameters in total.

The probabilistic model of $(E_{\mathrm{c}}, f_{\mathrm{cu}}, \varepsilon_{\mathrm{cu}}, \alpha_{\mathrm{c}})$ proposed by Refs. (Tao et al., 2020; Tao et al., 2021) is adopted. The marginal distribution of α_{c} follows the original model. For E_{c}, f_{cu}, and $\varepsilon_{\mathrm{cu}}$, the coefficients of variation are assumed to be independent of the concrete strength grade. Their distribution parameters are slightly adjusted to match the strength levels specified in the Chinese Code for Design of Concrete Structures (Ministry of Housing and Urban-Rural Development of the People's Republic of China, 2015). Their dependence structure also follows Ref. (Tao et al., 2020) where copulas are employed.

The tensile constitutive parameters $(f_{\mathrm{tu}}, \varepsilon_{\mathrm{tu}}, \alpha_t)$ follow the empirical relationships in Ref. (Guo et al., 1982) where they are expressed as deterministic functions of f_{tu}.

Based on the probabilistic model described above, 500 samples of the four compressive constitutive parameters are generated. The corresponding uniaxial compressive and tensile stress–strain curves for C30 concrete are shown in Fig. 2 for illustration. The remaining concrete properties, namely Poisson's ratio, biaxial strength factor, plastic hardening coefficient, and density, are taken as 0.2, 1.16, 0.1, and $2500\,\mathrm{kg/m^3}$, respectively.



(a) Compressive. (b) Tensile.

Figure 2. Samples of the uniaxial compressive and tensile stress–strain curves for C30 concrete.

3.3. Near-field seismic excitation

The structure is assumed to be subjected to seismic excitation acting along the Y-axis. The ground motion is modeled using a seismic ground motion model involving near-fault pulse-like ground motion. In this model, the acceleration, denoted by $\xi_{\mathrm{g}}(t)$, is described by (Jensen et al., 2012)

$$\xi_{\mathrm{g}}(t) = \mathcal{F}^{-1}\left\{\left[\left|\mathcal{F}\left(\xi_{\mathrm{g}}^{\mathrm{H}}(t)\right)\right| - \left|\mathcal{F}\left(\xi_{\mathrm{g}}^{\mathrm{L}}(t)\right)\right|\right] \exp\left(\mathrm{i}\cdot\arg\left\{\mathcal{F}\left[\xi_{\mathrm{g}}^{\mathrm{H}}(t)\right]\right\}\right)\right\} + \xi_{\mathrm{g}}^{\mathrm{L}}(t), \tag{29}$$

where $\xi_{\mathrm{g}}^{\mathrm{H}}(t)$ and $\xi_{\mathrm{g}}^{\mathrm{L}}(t)$ denote the high-frequency and low-frequency pulse-like components, respectively.

The high-frequency component $\xi_{\mathrm{g}}^{\mathrm{H}}(t)$ is generated by modulating a standard white noise $\xi_{\mathrm{w}}(t)$:

$$\xi_{\mathrm{g}}^{\mathrm{H}}(t) = \mathcal{F}^{-1}\left\{A(\omega; M, r)\,\frac{\mathcal{F}\{\psi(t; M, r)\cdot\xi_{\mathrm{w}}(t)\}}{\sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\mathcal{F}\{\psi(t; M, r)\}\right|^2 \mathrm{d}\omega}}\right\}, \tag{30}$$

where $\psi(t; M, r)$ and $A(\omega; M, r)$ denote the modulation function in the time and frequency domain, respectively. Both depend on the moment magnitude M and epicentral distance r. Further details of this relationship can be found in Ref. (Boore, 2003). The pulse-like component is described by the model proposed in Ref. (Mavroeidis and Papageorgiou, 2003)

$$\xi_{\mathrm{g}}^{\mathrm{L}}(t) = I\left\{t \in \left(t_{\mathrm{p}} - \frac{\gamma_{\mathrm{p}}}{2f_{\mathrm{p}}},\, t_{\mathrm{p}} + \frac{\gamma_{\mathrm{p}}}{2f_{\mathrm{p}}}\right)\right\} \cdot \; A_{\mathrm{p}}\left\{\sin\left[\frac{2\pi f_{\mathrm{p}}}{\gamma_{\mathrm{p}}}(t - t_{\mathrm{p}})\right]\cos\left[\frac{2\pi f_{\mathrm{p}}}{\gamma_{\mathrm{p}}}(t - t_{\mathrm{p}}) + \nu_{\mathrm{p}}\right] + \frac{2\pi f_{\mathrm{p}}}{\gamma_{\mathrm{p}}}\sin\left[\frac{2\pi f_{\mathrm{p}}}{\gamma_{\mathrm{p}}}(t - t_{\mathrm{p}}) + \nu_{\mathrm{p}}\right]\left[1 + \cos\left(\frac{2\pi f_{\mathrm{p}}}{\gamma_{\mathrm{p}}}(t - t_{\mathrm{p}})\right)\right]\right\}. \tag{31}$$

In this case, the moment magnitude M is set to 7.25. The epicentral distance r is considered as random. The parameters of the pulse model are also treated as random variables and their distributions are adopted from Ref. (Jensen et al., 2012).

Based on the probabilistic model described above, 500 samples of the pulse model are generated with a total duration of 25 s and a time step of 0.005 s. Owing to the variability of the epicentral distance r, the peak ground acceleration (PGA) also exhibits variability with a mean value of 2.81 $\mathrm{m/s^2}$ and a coefficient of variation of 0.26.

3.4. Probabilistic response of the structure under seismic ground motion

The proposed method is applied to determine the probabilistic response of the structure under the prescribed stochastic seismic excitation. The equation of motion of the FE model is given by Eq. (1), where $n = 80{,}472$, and $\boldsymbol{X}(t)$, $\dot{\boldsymbol{X}}(t)$, and $\ddot{\boldsymbol{X}}(t)$ denote the nodal displacement, velocity, and acceleration vectors, respectively. The excitation is taken as the stochastic ground acceleration $\xi_{\mathrm{g}}(t)$ in Eq. (29).

The vector $\boldsymbol{\Theta}$ collects the random material parameters in the concrete constitutive model, i.e., $\boldsymbol{\Theta} = (E_{\mathrm{c}}, f_{\mathrm{cu}}, \alpha_{\mathrm{c}}, \varepsilon_{\mathrm{cu}}, f_{\mathrm{tu}}, \alpha_{\mathrm{t}}, \varepsilon_{\mathrm{tu}})^{\top}$. When material plasticity, hysteresis, and damage accumulation are considered, the nonlinear restoring force depends not only on the current displacement and velocity but also on the internal variable $\boldsymbol{\eta}(t)$, which represents the material history. Therefore, $\boldsymbol{f}\left(\boldsymbol{X}(t), \dot{\boldsymbol{X}}(t), \boldsymbol{\eta}(t), t; \boldsymbol{\Theta}\right)$ is generally a functional rather than an explicit function and is written in the compact form given in Eq. (1).

The interstory drift angle of the sixth floor is selected as the quantity of interest and denoted by $X_{\mathrm{qoi}}(t)$. Its corresponding velocity is denoted by $V_{\mathrm{qoi}}(t) = \dot{X}_{\mathrm{qoi}}(t)$. As discussed in Section 2.2.2, the intrinsic diffusion functions of $X_{\mathrm{qoi}}(t)$ and $V_{\mathrm{qoi}}(t)$ vanish according to the probabilistic model of $\xi_{\mathrm{g}}(t)$. Therefore, the equation governing the auxiliary process is introduced as

$$\begin{cases} \mathrm{d}U_1(t) = U_2(t)\,\mathrm{d}t \\ \mathrm{d}U_2(t) = -\left[\omega_0^2(\boldsymbol{\Theta})\,U_1(t) + 2\zeta_0\omega_0(\boldsymbol{\Theta})\,U_2(t)\right]\mathrm{d}t + \sqrt{D}\xi_{\mathrm{g}}^{\mathrm{L}}(t)\,\mathrm{d}t + \sqrt{D}\mathrm{d}W(t)\,. \end{cases} \tag{32}$$

In the SDE, $\xi_{\mathrm{g}}^{\mathrm{L}}(t)$ denotes the low-frequency pulse component in Eq. (31). The Wiener process $W(t)$ corresponds to the standard white noise $\xi_{\mathrm{w}}(t)$ used in Eq. (30). Therefore, the transformation $\mathcal{G}$ in Eq. (21) is represented by the ground-motion model in Eq. (29). The natural circular frequency ω_0 is considered to depend on the random material properties and is specified as $\omega_0(\boldsymbol{\Theta}) = \omega_1(\boldsymbol{\Theta})/1.5$, where $\omega_1(\boldsymbol{\Theta})$ is the first natural circular frequency of the FE model. The damping ratio and diffusion intensity are set to $\zeta_0 = 0.05$ and $D = 0.1$, respectively. The process $U_2(t)$ is selected as the auxiliary diffusion process.

Next, the interstory drift angle of the sixth floor, $X_{\mathrm{qoi}}(t)$, is combined with the auxiliary diffusion process $U_2(t)$. The intrinsic drift functions of the joint process are estimated from 500 deterministic analyses using the LOWESS technique. The resulting two-dimensional DR-PDEE is then solved by the PIS scheme. The PDFs of the interstory drift angle at representative time instants are shown in Fig. (3).

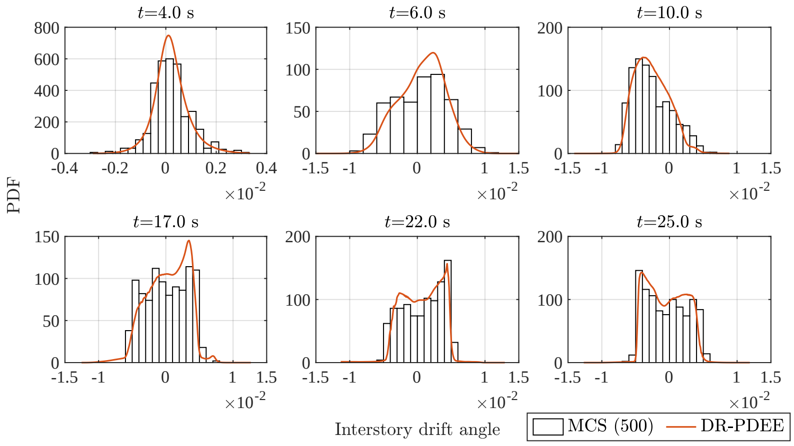


Figure 3. PDFs of the interstory drift angle of the structure at representative time instants.

The response distribution exhibits pronounced nonstationarity, skewness, and non-Gaussianity. Before the arrival of the low-frequency pulse, the PDF is unimodal. It gradually evolves into a bimodal distribution after the pulse. This evolution results from the combined effects of random material properties, structural nonlinearity, and uncertain excitation parameters. The bimodal feature becomes evident after the pulse peak, indicating an increasing influence of material nonlinearity. The proposed method captures this complex evolution. Despite the limited sample size imposed by the high cost of the deterministic analyses, the predicted PDFs agree well with the statistical estimates obtained from the 500 response samples.

4. Concluding Remarks

In this study, an efficient method is proposed for the stochastic response analysis of high-dimensional nonlinear systems under non-white excitation. The key idea is to formulate a low-dimensional DR-PDEE by incorporating an auxiliary diffusion process. A large-scale nonlinear RC structure subjected to pulse-like near-fault ground motions is investigated to demonstrate the effectiveness of the proposed method. The main conclusions are as follows:

1. By constructing the auxiliary diffusion process, a low-dimensional DR-PDEE with nonzero diffusion can be formulated without constructing a linear filter. This makes the DR-PDEE framework applicable to complex engineering excitations.

2. The auxiliary diffusion process can be constructed by generating white noise correlated with the original excitation and using it to drive a simple stochastic system. To improve the effectiveness

of the method, the selected auxiliary process should be strongly correlated with the velocity of the response of interest.

3. The engineering example demonstrates that the proposed method can quantitatively capture the time evolution of the response PDF of a large-scale nonlinear structure under stochastic near-fault ground motions. The method provides a theoretical basis for refined safety assessment of practical engineering structures.

Future work will focus on the optimal design of the governing equation for the auxiliary process and further develop the proposed method for reliability analysis.

Acknowledgements

The financial support from the National Natural Science Foundation of China (Grant Nos. 12472019 & 52494962) and the Chinesisch-Deutsches Zentrum für Wissenschaftsförderung (Grant No. M-0737) are highly appreciated.

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