Uncertainty Analysis of Fatigue Failure Using a Fuzzy Approach

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Abstract: Structural systems subjected to repeated or fluctuating loads are vulnerable to fatigue failure. This is because each cycle of loading causes damage to the system. Damage accumulates, causing cracks which reduce constituent member cross-sections, leading to an increased possibility of fracture. This can occur even when stresses remain below the yield limit. Fatigue failure is a multi-phase process that starts with a crack initiation phase and continues into a crack propagation phase.

In the conventional crack-initiation approach using Miner's rule, it is assumed that each cycle contributes a small increment of damage until failure occurs. This method relies on Stress–Life (S–N) curves derived from experiments. However, this approach is deterministic and does not reflect measurement imprecision or inherent uncertainties. These uncertainties derive from, material properties, geometry, fabrication quality, and cyclic loading. Uncertainty significantly affects the fatigue life of the system. Probabilistic and interval approaches have previously been used to enumerate these uncertainties. However, the former requires a well-defined distribution, whereas the latter requires exact bounds.

This work develops a new crack-initiation prediction method that uses a fuzzy approach for quantifying uncertainties in stress ranges as well as fatigue parameters. First, the fatigue coefficient and stress ranges are defined as fuzzy variables. These fuzzy variables are evaluated at multiple discrete levels of presumption, (α-cuts). Then, for each cut, the interval values of those variables are used to calculate the interval of damage and interval of fatigue life. Following that, the fuzzy values for interval damage and fatigue life are constructed. A numerical example is presented to demonstrate the applicability of the proposed method and it is compared with results obtained by Monte Carlo simulations. It is shown that obtaining fuzzy bounds on a system's fatigue life using this method does not require iterative methods.

Keywords: Fatigue, Damage, Uncertainty, Fuzzy

1. Introduction

Structural elements subject to repeated loading are susceptible to fatigue failure. This mode of failure can be attributed to the accumulation of damage at each cycle of loading. This damage accumulation over time can result in fracture even when stress levels remain below design limits. Fatigue failure is commonly described as a two-stage process which begins with crack initiation and followed by crack growth leading to final fracture. For analysis of these phases, a range of analytical methods has been developed to assess fatigue behavior and predict structural performance. For the first phase, the crack initiation method is widely used which is based on Miner's damage rule (Miner, 1945). This method assumes each load cycle contributes to the development of an infinitesimal amount of damage.

As the damage accumulates and reaches a critical value, fatigue failure occurs. Using this method, the fatigue life is estimated using the experimental relationships between the stress range and the number of cycles to failure. This is commonly expressed as an S–N curve (Basquin, 1910).

Conventional deterministic fatigue life prediction methods do not incorporate variations and uncertainty in the analysis of fatigue failure. These uncertainties can arise from numerous factors including scatter in laboratory fatigue data, variability in material and geometric properties, inaccuracies in field stress measurements, and modeling assumptions. The presence of these uncertainties may significantly affect predicted fatigue damage and remaining life, particularly when available data are limited. In order to incorporate the presence of uncertainties in the fatigue failure analysis, probabilistic approaches have been conventionally utilized (Ang and Munse, 1975; Sen, 2006; Kwon et al., 2012; Diekfuss and Foley, 2016). Furthermore, possibilistic approaches have been used that utilize interval analysis for fatigue life prediction (Desch and Modares, 2021**)** through which, uncertainties in fatigue parameters (from laboratory data) and stress ranges (from field measurements) are represented as interval variables. However, using probabilistic methods, the predicted fatigue analysis results are highly dependent on the assumption of the underlying probability distribution functions. On the other hand, possibilistic approaches, such as interval analysis, provide no information of the distribution of data.

In order to address the shortcomings of the aforementioned approaches, a fuzzy method can be utilized. Fuzzy set theory, first introduced by Zadeh (1965), provides a framework for representing imprecise information through membership functions. Fuzzy-based approaches are well suited for fatigue problems where limited data and modeling assumptions are major sources of uncertainty. Fuzzy approaches have been applied to fatigue analysis in which, imprecise information in the inputs is used to produce bounded predictions of fatigue damage, and fatigue life is represented without solely relying on probabilistic assumptions or possibilistic bounds. Muc and Kędziora (2001) used fuzzy sets to represent scatter in experimental fatigue and fracture data for composite materials. Tan et al. (2005) proposed a fuzzy reliability approach to predict the corrosion-fatigue life of aircraft structures. Zhang (2015) developed a fatigue life prediction model based on residual strength and cumulative damage concepts. However, work on fuzzy fatigue analysis has not yet produced a general framework to incorporate the uncertainties and variations from the experimental data level especially when limited data is available.

In this work, a new method is developed that is capable of determining the fatigue life of a structure using a fuzzy approach. In this method, fuzzy variables are utilized to quantify the uncertainties in the fatigue parameters (obtained from laboratory test data) as well as the stress ranges (obtained from field test data). Then, the fuzzy existing damage is determined, leading to a prediction of the structure's fuzzy remaining fatigue life. It is envisioned that the results obtained using this method can be used for reliability, safety, and management of aging infrastructure.

2. Background

This work is a symbiosis of two historically independent fields, fatigue life prediction and fuzzy analysis. In order to introduce the method, first, the background for both fields is presented.

2.1 Fatigue Life Prediction

The fatigue behavior of a structural component is determined using laboratory fatigue test data. The relationship between the stress range S and the number of cycles to failure N is expressed by the S–N relation as (Basquin, 1910):

$$N = \frac{C}{S^m} \tag{1}$$

where C is the fatigue coefficient and m is the fatigue exponent. These parameters are determined experimentally. The value of N represents the number of cycles to failure when the component is subjected to a constant stress range S. As experiments for the S-N relationship possess considerable scatter, data regression is used to calculate values for the parameters C and m.

Eq. 1 is defined for a single stress range; however, in actual service conditions, structural components experience different stress ranges throughout their lifetime. To account for the existence of multiple stress ranges, a cumulative damage method can be applied. As such, each load cycle produces a small amount of damage and over time, these damages are accumulated. The cumulative damage relationship is expressed as (Miner, 1945):

$$D = \sum \frac{n_i}{N_i} \tag{2}$$

where, n_i is the number of cycles applied at stress range S_i, and N_i is the number of cycles to failure at that stress range. According to Miner's rule, fatigue failure occurs when the total accumulated damage reaches the critical value of unity ($D = 1$). Combining Eqs. 1 and 2, the accumulated fatigue damage is written as:

$$D = \frac{1}{C} \cdot \sum_i n_i \cdot S_i^m \tag{3}$$

In order to obtain the stress ranges from a measured stress-time history, a cycle counting method can be used. The rainflow-counting algorithm is one cycle counting methods that is widely used in fatigue analysis (Matsuishi and Endo, 1968).

2.1.1 Regression of S–N Data

The fatigue parameters, C and m, are obtained from laboratory S-N data using regression analysis. First, both variables S and N are transformed into logarithmic space as $\ln S$ and $\ln N$ through which, this the nonlinear S–N relation becomes linear as:

$$\ln N = \ln C - m \ln S \tag{4}$$

Then, using least-squares regression, the fatigue coefficient and fatigue exponent are calculated as:

$$C = \exp\left(\frac{(\Sigma \ln N)(\Sigma (\ln S)^2) - (\Sigma \ln S)(\Sigma \ln S \ln N)}{p(\Sigma(\ln S)^2) - (\Sigma \ln S)^2}\right) \tag{5}$$

$$m = -\frac{p(\Sigma \ln S \ln N) - (\Sigma \ln S)(\Sigma \ln N)}{p(\Sigma(\ln S)^2) - (\Sigma \ln S)^2} \tag{6}$$

where, p is the number of data points. Moreover, using the least-squares method, the difference between the measured value and the predicted value for each data point, is expressed as:\

$$e_j = \ln N_j - \ln C + m \ln S_j \tag{7}$$

where, e_j is the residual value.

2.2 UNCERTAINTY IN FATIGUE LIFE PREDICTION

These residual values obtained in Eq. 7 depict the scatter in experimental data which can be considered as indicatives of the presence of uncertainty in the estimated fatigue parameters. Since the uncertainties in the fatigue parameters are fully correlated (Ang and Munse, 1975), they can be represented through a single uncertain variable. Accordingly, the uncertainty can be solely considered in the fatigue parameter, *C*, while the fatigue exponent, *m*, is considered deterministic (Desch and Modares 2021).

2.3 FUZZY REPRESENTATION OF UNCERTAINTY

Fuzzy set theory is a possibilistic approach to quantify uncertainty. In fuzzy analysis, an uncertain parameter is described using a fuzzy variable. In this work, the fuzzy variables are depicted in bold.

2.3.1 FUZZY REPRESENTATION OF UNCERTAINTY

Considering *E* as a referential set, in an ordinary subset the characteristic function is:

$$\forall x \in E: \mu_A(x) \in \{0,1\} \tag{8}$$

In a fuzzy subset the characteristic function (membership function) is:

$$\forall x \in E: \mu_A(x) \in [0,1] \tag{9}$$

which allows for partial membership (Figure 1).

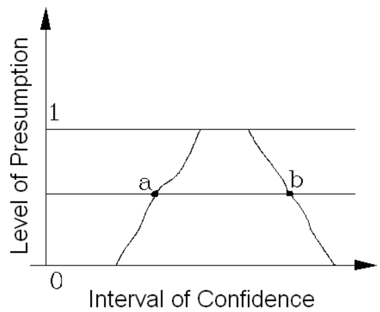

Figure 1. The membership function of a fuzzy variable

For each level of presumption (α-cut), there exists a corresponding level of confidence such that:

$$\forall \alpha_1, \alpha_2 \in [0,1], \alpha_1 < \alpha_2; [a^{\alpha_2}, b^{\alpha_2}] \subset [a^{\alpha_1}, b^{\alpha_1}] \quad (10)$$

2.3.2 Triangular Fuzzy Numbers

One of the most commonly used fuzzy numbers is the triangular fuzzy variable (Figure 2).

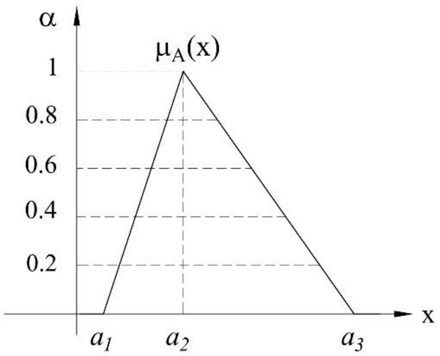

Figure 2. The membership function of a triangular fuzzy variable

A triangular fuzzy variable is defined using three values:

$$\boldsymbol{A} = (a_1,\ a_2,\ a_3) \quad (11)$$

where, a_1 is the lower bound, a_2 is the most likely value and a_3 is the upper bound. The α-cut of a triangular fuzzy variable is given by:

$$\boldsymbol{A_\alpha} = [a_1 + (a_2 - a_1)\alpha, a_3 - (a_3 - a_2)\alpha] \quad (12)$$

At $\alpha = 0$, the interval is $[a_1, a_3]$, representing the maximum uncertainty, whereas at $\alpha = 1$, the interval of confidence reduces to the single value a_2, representing the deterministic case.

2.3.3 Basic Operations on Fuzzy Variables

Considering two fuzzy variables $\boldsymbol{A}$ and $\boldsymbol{B}$ be represented at the α-cut level through their intervals of confidence as $\boldsymbol{A_\alpha} = [\underline{A_\alpha}, \overline{A_\alpha}]$ and $\boldsymbol{B_\alpha} = [\underline{B_\alpha}, \overline{B_\alpha}]$, respectively, the basic interval arithmetic operations are:

Addition:

$$\boldsymbol{A_\alpha} + \boldsymbol{B_\alpha} = \left[\underline{A_\alpha} + \underline{B_\alpha}, \overline{A_\alpha} + \overline{B_\alpha}\right] \quad (13)$$

Subtraction:

$$\boldsymbol{A_\alpha} - \boldsymbol{B_\alpha} = \left[\underline{A_\alpha} - \overline{B_\alpha}, \overline{A_\alpha} - \underline{B_\alpha}\right] \quad (14)$$

Multiplication:

$$\boldsymbol{A_\alpha} \times \boldsymbol{B_\alpha} = [\min(\{AB\}), \max(\{AB\})] \tag{15}$$

where:

$$AB = \left\{\left(\underline{A_\alpha} \cdot \underline{B_\alpha}\right), \left(\underline{A_\alpha} \cdot \overline{B_\alpha}\right), \left(\overline{A_\alpha} \cdot \underline{B_\alpha}\right), \left(\overline{A_\alpha} \cdot \overline{B_\alpha}\right)\right\} \tag{16}$$

Division:

$$\boldsymbol{A_\alpha} \div \boldsymbol{B_\alpha} = \left[\min\left(\left\{\frac{A}{B}\right\}\right), \max\left(\left\{\frac{A}{B}\right\}\right)\right] \tag{17}$$

where:

$$\frac{A}{B} = \left\{\left(\underline{A_\alpha}/\underline{B_\alpha}\right), \left(\underline{A_\alpha}/\overline{B_\alpha}\right), \left(\overline{A_\alpha}/\underline{B_\alpha}\right), \left(\overline{A_\alpha}/\overline{B_\alpha}\right)\right\} \tag{18}$$

and where, $0 \notin \boldsymbol{B_\alpha}$.

3. Methodology

3.1 Formulation of Fuzzy Fatigue Failure Analysis

The general algorithm and major steps for fuzzy fatigue failure analysis and fatigue life prediction are outlined as follows.

Step 1. Construct a Fuzzy S–N Relationship from laboratory test data

In this step, first, the fuzzy fatigue coefficient, $\boldsymbol{C}$, is constructed based on a triangular membership function. For α = 0, the laboratory test data is deterministically regressed and the corresponding interval of confidence for fuzzy fatigue coefficient $\boldsymbol{C}$ is:

$$\boldsymbol{C_{\alpha=0}} = [\underline{C}, \overline{C}] = C \cdot [\delta_{C_l}, \delta_{C_u}] \tag{19}$$

where, C is the deterministic fatigue coefficient and δ_{C_l} and δ_{C_u} are the lower and upper variations, respectively; obtained by an enveloping procedure as:

$$\delta_{C_l} = \exp\left(\min\left(e_j\right)\right) \tag{20}$$

$$\delta_{C_u} = \exp\left(\max\left(e_j\right)\right) \tag{21}$$

For α = 1, the deterministic fatigue coefficient C is considered as the most likely value. Then, the fuzzy fatigue coefficient is constructed through its triangular membership function as:

$$\boldsymbol{C} = \left\{0\ (c \le \underline{C}), \frac{c-\underline{C}}{C-\underline{C}}(\underline{C} \le c \le C), \frac{\overline{C}-c}{\overline{C}-C}(C \le c \le \overline{C}), 0\ (c \ge \overline{C})\right\} \tag{22}$$

where, the corresponding α-cut for the fuzzy fatigue coefficient is:

$$\boldsymbol{C_\alpha} = [\underline{C} + (C - \underline{C})\alpha, \overline{C} - (\overline{C} - C)\alpha] \tag{23}$$

Figure 3 depicts fuzzy fatigue coefficient schematically.

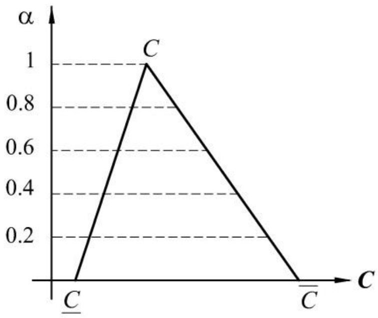

Figure 3. Fuzzy Fatigue Coefficient

By considering the fatigue exponent, *m*, as deterministic (i.e., the uncertainty is enumerated in fuzzy fatigue coefficient $\boldsymbol{C}$), the fuzzy S-N relationship can be constructed. Figure 4 depicts the interval of confidence for α = 0 of the fuzzy S-N relationship schematically.

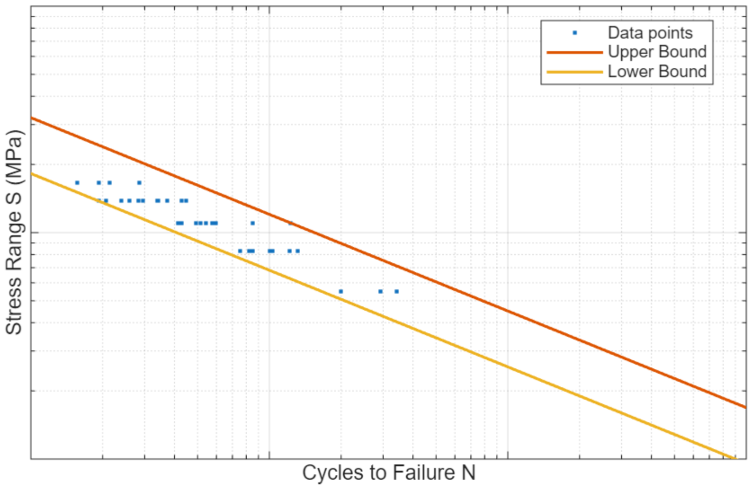

Figure 4. Interval of confidence for α = 0 of the fuzzy S-N relationship

Step 2. Determine Fuzzy Stress Ranges and Cycle Counts from Field Data

In this step, the fuzzy stress ranges are constructed using obtained field data followed by a cycle counting procedure. For α = 0 the interval of confidence for each stress range, i, is:

$$\boldsymbol{S}_{\boldsymbol{i}_{\boldsymbol{\alpha}=\boldsymbol{0}}} = \left[\underline{S_i}, \overline{S_i}\right] = \left[S_i - \delta_{S_{i_l}}, S_i + \delta_{S_{i_u}}\right] \tag{24}$$

where, S_i is the deterministic stress range, $\underline{S_i}$ and $\overline{S_i}$ are the lower and upper bounds of the stress range, and $\delta_{S_{i_l}}$ and $\delta_{S_{i_u}}$ are the variation parameters for lower and upper bounds of the stress range, respectively. The variation parameters can be obtained by (a) the tolerance of sensors used in field data collection, (b) the level of rounding used in the cycle-counting process, and/or (c) engineering judgment or expert opinion. For α = 1, the deterministic stress range S_i is considered as most likely value.

Then, for each stress range, the fuzzy stress range is constructed through its triangular membership function as:

$$\boldsymbol{S}_{\boldsymbol{i}} = \left\{0\ \left(s \le \underline{S_i}\right), \frac{s - \underline{S_i}}{S_i - \underline{S_i}}\left(\underline{S_i} \le s \le S_i\right), \frac{\overline{S_i} - s}{\overline{S_i} - S_i}\left(S_i \le s \le \overline{S_i}\right), 0\ \left(s \ge, \overline{S_i}\right)\right\} \tag{25}$$

The corresponding α-cut for the fuzzy stress range is:

$$\boldsymbol{S}_{\boldsymbol{i}_{\boldsymbol{\alpha}}} = \left[\underline{S_i} + \left(S_i - \underline{S_i}\right)\alpha, \overline{S_i} - \left(\overline{S_i} - S_i\right)\alpha\right] \tag{26}$$

Step 3. Determine the Fuzzy Damage Accumulated over the Duration of Field Data Collection

At a given level α, using the corresponding intervals of confidence for fuzzy fatigue coefficient $\boldsymbol{C}_{\boldsymbol{\alpha}}$ and fuzzy stress ranges $\boldsymbol{S}_{\boldsymbol{i}_{\boldsymbol{\alpha}}}$, the interval of confidence for fuzzy accumulated damage over the duration of the field data collection, is calculated as:

$$\boldsymbol{D}_{\boldsymbol{d}_{\boldsymbol{\alpha}}} = \frac{1}{\boldsymbol{C}_{\boldsymbol{\alpha}}} \sum_i n_i \left(\boldsymbol{S}_{\boldsymbol{i}_{\boldsymbol{\alpha}}}\right)^m \tag{27}$$

or equivalently:

$$\boldsymbol{D}_{\boldsymbol{d}_{\boldsymbol{\alpha}}} = \frac{1}{\left[\underline{C} + \left(C - \underline{C}\right)\alpha, \overline{C} - \left(\overline{C} - C\right)\alpha\right]} \sum_i n_i \left(\underline{S_i} + \left(S_i - \underline{S_i}\right)\alpha, \overline{S_i} - \left(\overline{S_i} - S_i\right)\alpha\right)^m \tag{28}$$

Then, using various α-cut levels, the fuzzy accumulated damage over the duration of the field data collection $\boldsymbol{D}_{\boldsymbol{d}}$ is constructed.

Step 4. Calculation of Fuzzy Mean Damage Rate, Fuzzy Existing Damage, Fuzzy Fatigue Life, and Fuzzy Remaining Life

At a given level α:

1. The interval of confidence for fuzzy mean damage is calculated as:

$$\boldsymbol{D}_{\boldsymbol{m}_{\boldsymbol{\alpha}}} = \frac{\boldsymbol{D}_{\boldsymbol{d}_{\boldsymbol{\alpha}}}}{t_d} = \left[\frac{\underline{\boldsymbol{D}_{\boldsymbol{d}_{\boldsymbol{\alpha}}}}}{t_d}, \frac{\overline{\boldsymbol{D}_{\boldsymbol{d}_{\boldsymbol{\alpha}}}}}{t_d}\right] \tag{29}$$

where, t_d is the duration of field data collection.

2. The interval of confidence for fuzzy existing damage is calculated as:

$$\boldsymbol{D}_{\boldsymbol{e}_{\boldsymbol{\alpha}}} = \boldsymbol{D}_{\boldsymbol{m}_{\boldsymbol{\alpha}}}.t_a = \left[\underline{\boldsymbol{D}_{\boldsymbol{m}_{\boldsymbol{\alpha}}}}.t_a, \overline{\boldsymbol{D}_{\boldsymbol{m}_{\boldsymbol{\alpha}}}}.t_a\right] \tag{30}$$

where t_a is the current age of the structure.

3. The interval of confidence for fuzzy fatigue life is calculated as:

$$\boldsymbol{t}_{\boldsymbol{l}_{\boldsymbol{\alpha}}} = \frac{1}{\boldsymbol{D}_{\boldsymbol{m}_{\boldsymbol{\alpha}}}} = \left[\frac{1}{\overline{\boldsymbol{D}_{\boldsymbol{m}_{\boldsymbol{\alpha}}}}}, \frac{1}{\underline{\boldsymbol{D}_{\boldsymbol{m}_{\boldsymbol{\alpha}}}}}\right] \tag{31}$$

4. The interval of confidence for fuzzy remaining fatigue life calculated as:

$$\boldsymbol{t}_{\boldsymbol{r}_{\boldsymbol{\alpha}}} = \boldsymbol{t}_{\boldsymbol{l}_{\boldsymbol{\alpha}}} - t_a = \left[\underline{\boldsymbol{t}_{\boldsymbol{l}_{\boldsymbol{\alpha}}}} - t_a, \overline{\boldsymbol{t}_{\boldsymbol{l}_{\boldsymbol{\alpha}}}} - t_a\right] \tag{32}$$

Following that, using various α-cut levels, fuzzy mean damage $\boldsymbol{D}_{\boldsymbol{m}}$, fuzzy existing damage $\boldsymbol{D}_{\boldsymbol{e}}$, fuzzy fatigue life $\boldsymbol{t}_{\boldsymbol{l}}$, and fuzzy remaining fatigue life $\boldsymbol{t}_{\boldsymbol{r}}$ are constructed.

4. Numerical Example

4.1 Problem Definition

In this example, the developed method is applied to determine the existing damage and remaining fatigue life of a steel bridge component subjected to cyclic loading. Moreover, the main results are compared with 10^4 Monte Carlo simulation realizations using uniformly distributed random variables.

The age of the bridge under consideration is considered to be 20 years. The structure is fabricated from A514 structural steel. The most fatigue-critical details are identified as the cover plates attached to the bottom flanges of the deck girders, with un-welded ends, as shown in the original study (Figure 5). Laboratory fatigue test data for this detail and material combination are obtained from Fisher et al. (1969) (Table I).

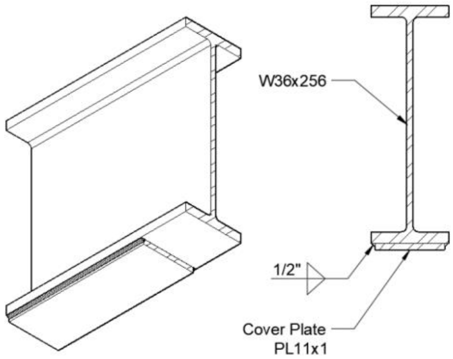

Figure 5. Cover plate detail, ends un-welded

Table I: Laboratory S-N data from Fisher et al. 1969												
S_i		N (load Cycles ×10^3)										
MPa	ksi	Multiple tests										
55.16	8	1988.9	2916.2	3409.2								
82.74	12	1031.1	848.3	1310.9	821.7	1004.7	1220.0	755.2				
110.32	16	514.8	1227.8	854.9	428.5	542.2	598.5	429.9	412.5	589.6	578.0	
137.9	20	341.3	429.1	445.9	282.3	192.3	339.5	260.0	238.8	374.0	296.0	207.0
165.47	24	156.6	213.8	285.2	192.5							

Field stress range measurements and corresponding cycle counts for bridge #0160335 are obtained from monitoring data reported by Hahin et al. (1993) (table II). The cycle counts correspond to a 24-hour data collection period.

Table II: Stress range & cycle count data Hahin et al. 1993		
S_i		n_i
MPa	ksi	Cycles
6.89	1.0	2843
10.3	1.5	991.0
13.79	2.0	386
17.24	2.5	111
20.68	3.0	88
24.13	3.5	64
27.58	4.0	33
31.03	4.5	32
34.47	5.0	15
37.92	5.5	7

4.2 PROBLEM SOLUTION:

Step 1. In this step the fatigue parameters of the fuzzy S-N relationship is determined. First, the laboratory test data are regressed. Then, for the intervals of fuzzy fatigue coefficient for various α-levels are determined. Moreover, fatigue exponents are considered deterministic (Table III and Figure 6).

Table III: Fuzzy fatigue coefficient and fatigue exponent

α	Fuzzy Fatigue coefficient (***C***)		Fatigue Exponent (*m*)
	$Mpa^{m}*10^{10}$	$ksi^{m}*10^{8}$	
0	[1.976,7.481]	[2.148,8.132]	2.343
0.2	[2.231,6.635]	[2.424,7.211]	2.343
0.4	[2.485,5.788]	[2.701,6.291]	2.343
0.6	[2.739,4.941]	[2.977,5.371]	2.343
0.8	[2.993,4.094]	[3.253,4.450]	2.343
1	[3.247,3.247]	[3.530,3.530]	2.343

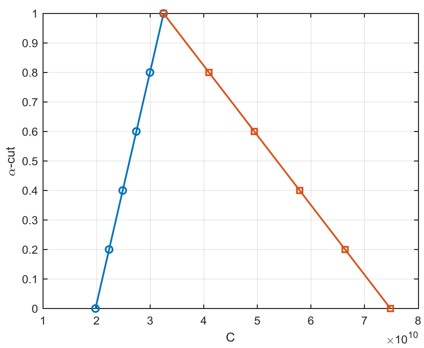

Figure 6. Fuzzy fatigue coefficient (***C***)

Step 2. In this step, the fuzzy stress ranges are determined. The stress ranges obtained from field monitoring are used to construct symmetric isosceles triangular membership functions. At α = 0, ±1% uncertainty is considered, while at α = 1 stress range S_i is considered deterministic. The results for various stress ranges are also obtained (Tables IV-a through IV-f.)

Table IV-a. Fuzzy stress range S_i, α=0	
MPa	ksi
[6.82,6.96]	[0.99,1.01]
[10.24,10.44]	[1.48,1.51]
[13.65,13.93]	[1.98,2.02]
[17.07,17.41]	[2.47,2.52]
[20.47,20.89]	[2.97,3.03]
[23.89,24.37]	[3.46,3.53]
[27.30,27.86]	[3.96,4.04]
[30.72,31.34]	[4.45,4.54]
[34.13,34.81]	[4.95,5.05]
[37.54,38.30]	[5.44,5.55]

Table IV-b. Fuzzy stress range S_i, α = 0.2	
Mpa	ksi
[6.835,6.945]	[0.991,1.007]
[10.257,10.423]	[1.487,1.511]
[13.680,13.900]	[1.984,2.016]
[17.102,17.378]	[2.480,2.520]
[20.515,20.845]	[2.975,3.023]
[23.937,24.323]	[3.471,3.527]
[27.359,27.801]	[3.967,4.031]
[30.782,31.278]	[4.463,4.535]
[34.194,34.746]	[4.958,5.038]
[37.617,38.223]	[5.454,5.542]

Table IV-c. Fuzzy stress range S_i, α = 0.4	
Mpa	Ksi
[6.849,6.931]	[0.993,1.005]
[10.278,10.402]	[1.490,1.508]
[13.707,13.873]	[1.988,2.0212]
[17.137,17.343]	[2.485,2.515]
[20.556,20.804]	[2.981,3.017]
[23.985,24.275]	[3.478,3.520]
[27.415,27.745]	[3.975,4.023]
[30.844,31.216]	[4.472,4.526]
[34.263,34.677]	[4.968,5.028]
[37.692,38.148]	[5.465,5.531]

Table IV-d. Fuzzy stress range S_i, α = 0.6	
Mpa	ksi
[6.862, 6.918]	[0.995,1.003]
[10.299, 10.381]	[1.493,1.505]
[13.735, 13.845]	[1.992,2.008]
[17.171, 17.309]	[2.490,2.510]
[20.597, 20.763]	[2.987,3.011]
[24.033, 24.227]	[3.485,3.513]
[27.470, 27.690]	[3.983,4.015]
[30.906, 31.154]	[4.481,4.517]
[34.332, 34.608]	[4.978,5.018]
[37.768, 38.072]	[5.476,5.520]

Table IV-e. Fuzzy stress range S_i, α = 0.8	
Mpa	Ksi
[6.876,6.904]	[0.997,1.001]
[10.319,10.361]	[1.496,1.502]
[13.762,13.818]	[1.996,2.004]
[17.206,17.274]	[2.495,2.505]
[20.639,20.721]	[2.996,3.005]
[24.082,24.178]	[3.492,3.506]
[27.525,27.635]	[3.991,4.007]
[30.968,31.092]	[4.490,4.508]
[34.401,34.539]	[4.988,5.008]
[37.844,38.996]	[5.487,5.509]

Table IV-f. Fuzzy stress range S_i, α = 1	
Mpa	ksi
6.890	0.999
10.340	1.499
13.790	2.000
17.240	2.500
20.680	2.999
24.130	3.499
27.580	3.999
31.030	4.499
34.470	4.998
37.920	5.498

Step 3. In this step, the accumulated fuzzy fatigue damage over the one-day field data collection period ($t_d = 1/365$ years) is determined and compared with Monte-Carlo simulation results (Table V and Figure 7).

Table V. Fuzzy damage during one day of data collection $\boldsymbol{D_d}$ ($\times 10^{-5}$)		
α	Fuzzy Results	MCS Results
0	[1.6384,6.499]	[1.7173,6.2900]
0.2	[1.856,5.732]	[1.9291,5.6090]
0.4	[2.138,5.122]	[2.2011,4.9954]
0.6	[2.516,4.625]	[2.5653,4.5543]
0.8	[3.051,4.212]	[3.0799,4.2030]
1	[3.865,3.865]	[3.8645,3.8645]

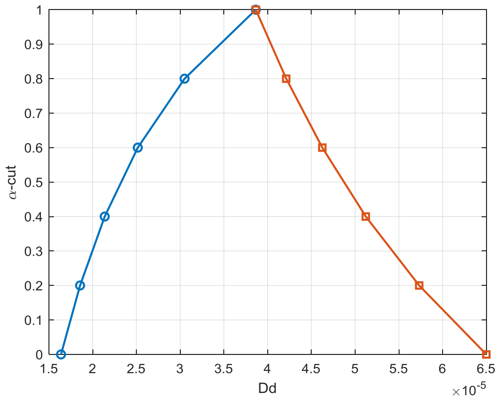

Figure 7. Fuzzy damage for one day of data collection

Step 4. In this step, the fuzzy mean damage rate, fuzzy existing damage ($t_a = 20$ years), fuzzy fatigue life, and fuzzy remaining life are determined (Tables VI-VIII and Figures 8-10).

Table VI: Existing Damage Results $\boldsymbol{D_e}$		
α	Fuzzy Results	MCS Results
0	[0.1196,0.47449]	[0.12536,0.45917]
0.2	[0.13551,0.41846]	[0.14083,0.40946]
0.4	[0.15607,0.3739]	[0.16068,0.36466]
0.6	[0.18368,0.33762]	[0.18727,0.33246]
0.8	[0.22271,0.3075]	[0.22483,0.30682]
1	[0.28211,0.28211]	[0.28211,0.28211]

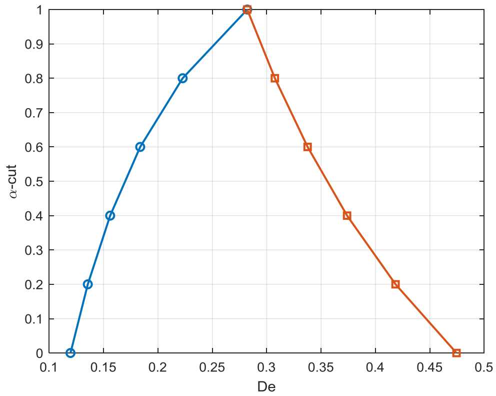

Figure 8. Fuzzy existing damage

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Table VII: Fatigue Life Results t_l (Years)		
α	Fuzzy Results	MCS Results
0	[42.15,167.22]	[43.56,159.54]
0.2	[47.79,147.59]	[48.84,142.02]
0.4	[53.49,128.15]	[54.84,124.47]
0.6	[59.24,108.89]	[60.16,106.8]
0.8	[65.04,89.8]	[65.19,88.96]
1	[70.89,70.89]	[0.89,0.89]

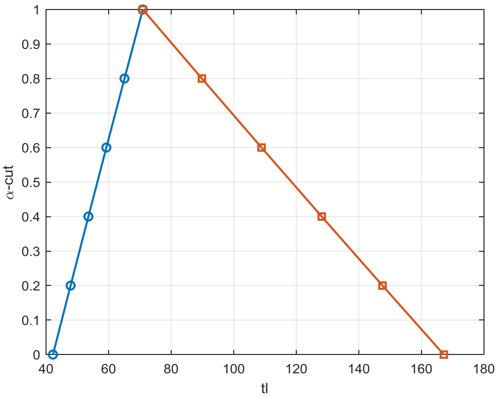

Figure 9. Fuzzy fatigue life t_l (years)

Table VIII: Remaining Life t_r (Years)		
α	Fuzzy Results	MCS Results
0	[22.15, 147.22]	[23.56,139.54]
0.2	[27.79, 127.59]	[28.84, 122.02]
0.4	[33.49, 108.15]	[34.84, 104.47]
0.6	[39.24, 88.89]	[40.16, 86.8]
0.8	[45.04, 69.8]	[45.19, 68.96]
1	50.89	50.89

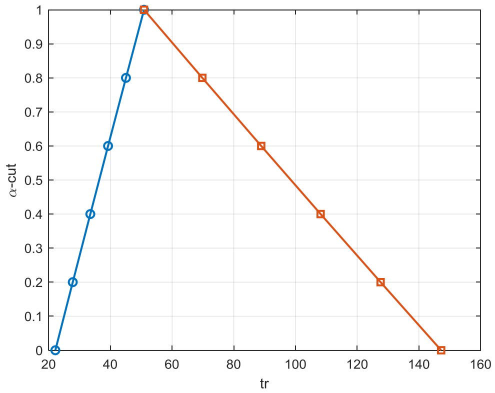

Figure 9. Fuzzy remaining fatigue t_r (years)

4.3 OBSERVATIONS

The results show that the predicted remaining fatigue life is strongly affected by the assumed level of uncertainty. For the lowest assumption level (α = 0), the remaining life varies widely, with a lower bound of 22.15 years and an upper bound of 147.22 years. This large range demonstrates the high sensitivity of fatigue life predictions to uncertainties in material properties, loading conditions, and field or laboratory measurements. As a result, greater uncertainty in input data leads to wider bounds in the estimated remaining life.

As anticipated, as the level of assumption decreases (i.e., with increasing α), uncertainty is reduced and the upper and lower bounds move closer to each other. This indicates increased confidence in the prediction. Additionally, the Monte Carlo simulation results for each α-cut are the inner bounds of the results by verifying the validity of the developed method.

5. Summary and Conclusions

In this paper, a new method for fatigue analysis of structures and determination of their fatigue life is developed that is capable of considering uncertainties using fuzzy analyses. This method includes uncertainties from both laboratory and field data, as well as uncertainties arising from the utilization of fuzzy variables and the construction of fuzzy functions. Due to the existence of various levels of presumption in the fuzzy approach it can produce results with more realistic consideration of uncertainties, when compared with an interval approach. The results are also compared with Monte Carlo simulations, which show the simulation results are inner bounds of the results obtained by the developed method. The simplicity and flexibility of this method, especially when data are limited, make it a useful tool for fatigue life prediction.

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