A Transformation-Based Framework for Modeling and Simulation of Multivariate Non-Gaussian Random Fields

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Abstract: Accurate modeling of multivariate non-Gaussian random fields remains a fundamental challenge in reliable engineering computation, particularly when nonlinear dependencies coexist with spatial correlation. This study introduces a new framework in which the probabilistic model is defined by the original marginal distributions together with linear correlations of Gaussianized transformed variables. This leads to a transformed correlation function that preserves non-Gaussian marginals while consistently characterizing spatial variability. The bridge function, originally developed for Gaussian fields, is extended to reconcile spatial correlation with nonlinear dependencies such as those captured by vine copulas. Based on this model, an efficient non-iterative simulation algorithm is developed. Numerical results show that the method accurately reproduces marginal distributions, nonlinear dependence, and spatial correlation in complex engineering fields.

Keywords: Multivariate non-Gaussian random field; Transformed correlation function; Bridge function; Uncertainty quantification.

1. Introduction

Reliable engineering computing and structural risk assessment increasingly rely on the rigorous quantification of spatial uncertainties (Li and Lyu, 2026). In civil, mechanical, and geotechnical engineering, physical parameters (such as material compressive strength, elastic modulus, and fracture toughness) rarely manifest as homogeneous or isolated random variables (Tao and Chen, 2023). Instead, they exhibit distinct spatial variability over the physical domain, which is mathematically characterized using random fields (Vanmarcke, 2010). In many engineering systems, multiple spatially varying quantities are physically coupled, necessitating the use of multivariate random field models (Tao et al., 2020). An accurate probabilistic representation of these multivariate fields requires the simultaneous capture of three essential characteristics (Lyu et al., 2025): (1) the individual marginal probability distributions of each variable, (2) the spatial auto-correlation governing the spatial fluctuation of each individual component, and (3) the multivariate joint dependence describing the point-wise and cross-spatial relationships among the different variables.

Traditionally, multivariate random fields are modeled using power spectral density (PSD) matrices or cross-correlation functions (Song et al., 2018). While these models are well-suited for multivariate Gaussian fields, they present significant limitations when applied to non-Gaussian variables (Grigoriu, 1995). Most realistic engineering parameters exhibit highly skewed, heavy-tailed, or bounded distributions (Zhang et al., 2026). When modeling these non-Gaussian variables, assuming joint normality is no longer appropriate. To address this, translation field models, such as Nataf-based models, are commonly utilized to map non-Gaussian fields into auxiliary Gaussian fields (Liu and Der Kiureghian, 1986). However, this isoprobability mapping inevitably distorts the original correlation structure. Reconciling the target spatial correlation of the non-Gaussian field with the correlation of the auxiliary Gaussian field is a known modeling challenge. Iterative schemes, such as those relying on Hermite polynomial expansions or iterative PSD adjustments, are frequently employed to correct these correlation distortions (Deodatis and Micaletti, 2001; Li and Xu, 2023). Nevertheless, these corrective procedures can be computationally demanding, and their convergence is not always guaranteed, particularly under highly non-Gaussian marginals or strong spatial gradients.

The challenge becomes even more pronounced when considering the between-variable dependencies in a multivariate setting (Vorechovsky 2008). Standard translation-based workflows typically assume that the underlying dependence can be adequately represented by linear Pearson correlation coefficients or Gaussian copulas. In real-world materials, however, inter-variable relationships are often highly nonlinear, asymmetric, or tail-dependent (Liu et al., 2024). For instance, concrete tensile strength and fracture energy may show asymmetric association under extreme values (Hai and Lyu, 2023). Although copula theory, particularly vine copulas, has gained traction for representing complex high-dimensional dependencies (Joe, 2014), a critical gap remains: existing copula models do not easily integrate with the spatial correlation structures of random fields without introducing modeling inconsistencies or requiring computationally expensive iterative simulation algorithms.

To address these challenges, this study presents a transformation-based framework for modeling and simulating multivariate non-Gaussian random fields. The proposed approach defines the spatial correlation of the non-Gaussian field directly through the linear correlation of its Gaussianized auxiliary variables, yielding what is termed the transformed correlation function (Liu et al., 2026). This representation is inherently compatible with arbitrary marginal distributions and bypasses the modeling incompatibility of traditional approaches. Furthermore, the *bridge function* concept, originally developed for multivariate Gaussian fields (Lyu et al., 2025), is extended to non-Gaussian systems to map spatial correlation and arbitrary copula-based nonlinear dependencies. Building upon this formulation, a non-iterative and efficient simulation algorithm is developed to generate realizations of multivariate non-Gaussian fields (Fu et al., 2026). The capability and accuracy of the proposed framework are demonstrated through practical engineering examples, showcasing its viability for reliable engineering computations.

The remainder of this paper is organized as follows. Section 2 introduces the concept and mathematical construction of the transformed correlation function for univariate non-Gaussian random fields. Section 3 details the multivariate modeling framework, focusing on the extended bridge function and its integration with vine copulas to describe complex, nonlinear between-variable dependencies. In Section 4, the non-iterative and computationally efficient simulation algorithms for multivariate non-Gaussian fields are presented. Section 5 validates the proposed framework using an engineering case study: a bivariate field of concrete tensile strength and fracture energy. Finally, Section 6 summarizes the key findings of this study and outlines potential directions for future research.

2. Concept of Transformed Correlation Function

To properly characterize a univariate homogeneous non-Gaussian random field $Y(\boldsymbol{x})$ with mean μ and standard deviation σ, one must model both its marginal distribution and its spatial variability. The marginal distribution is invariant with respect to the spatial position due to homogeneity and is typically represented by its cumulative distribution function (CDF), $F(y)$, or probability density function (PDF), $p(y)$. Spatial variability is traditionally characterized by the original correlation coefficient function $\rho(\boldsymbol{\xi})$, where $\boldsymbol{\xi}$ denotes the distance vector between two spatial coordinates. However, defining non-Gaussian spatial variability solely via $\rho(\boldsymbol{\xi})$ alongside $F(y)$ is statistically inconsistent at the modeling stage (Liu and Der Kiureghian, 1986). The classical Pearson correlation assumes linear dependence, which is strictly valid only in the Gaussian framework. Imposing non-Gaussian marginals on a field simulated with a target $\rho(\boldsymbol{\xi})$ inevitably distorts the spatial correlation, requiring tedious iterative corrections. To resolve this inconsistency, this section introduces the transformed correlation function as a statistically coherent descriptor of spatial variability.

First, the non-Gaussian field $Y(\boldsymbol{x})$ is mapped to an auxiliary standard Gaussian field $Z(\boldsymbol{x})$ using the isoprobability transformation $Z(\boldsymbol{x}) = \Phi^{-1}\{F[Y(\boldsymbol{x})]\}$, where $\Phi(\cdot)$ denotes the CDF of the standard normal distribution. Because this transformation is strictly monotonic, it preserves the rank-order structure of the random field. Consequently, $Z(\boldsymbol{x})$ is a homogeneous standard Gaussian field with zero mean and unit variance. The spatial correlation of $Z(\boldsymbol{x})$ is characterized by its correlation function $\rho_{\mathrm{G}}(\boldsymbol{\xi})$, which is defined as

$$\rho_{\mathrm{G}}(\boldsymbol{\xi}) = \mathrm{E}[Z(\boldsymbol{x})Z(\boldsymbol{x}+\boldsymbol{\xi})] = \mathrm{E}[\Phi^{-1}\{F[Y(\boldsymbol{x})]\}\Phi^{-1}\{F[Y(\boldsymbol{x}+\boldsymbol{\xi})]\}]. \tag{1}$$

In this study, $\rho_{\mathrm{G}}(\boldsymbol{\xi})$ is referred to as the *transformed correlation function* of $Y(\boldsymbol{x})$ (Liu et al., 2026). Because $Z(\boldsymbol{x})$ is a mathematically valid standard Gaussian field, its correlation function $\rho_{\mathrm{G}}(\boldsymbol{\xi})$ naturally satisfies all essential mathematical properties of a correlation function, such as symmetry, boundedness, and positive definiteness. Using $\rho_{\mathrm{G}}(\boldsymbol{\xi})$ to model spatial variability ensures modeling compatibility, as any Gaussian field simulated with $\rho_{\mathrm{G}}(\boldsymbol{\xi})$ can be transformed directly into the target non-Gaussian field $Y(\boldsymbol{x})$ without distorting the underlying spatial structure.

There exists a deterministic functional relationship between the original correlation coefficient function $\rho(\boldsymbol{\xi})$ and the transformed correlation function $\rho_{\mathrm{G}}(\boldsymbol{\xi})$. By modeling the joint probability of the auxiliary

Gaussian variables at two positions $\boldsymbol{x}$ and $\boldsymbol{x}+\boldsymbol{\xi}$ through a Gaussian copula, the correlation coefficient $\rho(\boldsymbol{\xi})$ can be derived using the joint PDF of $Y(\boldsymbol{x})$ and $Y(\boldsymbol{x}+\boldsymbol{\xi})$ as

$$\rho(\boldsymbol{\xi})=\frac{1}{\sigma^2}\int_0^1\int_0^1 F^{-1}(u_1)F^{-1}(u_2)c_{\mathrm{G}}[u_1,u_2;\rho_{\mathrm{G}}(\boldsymbol{\xi})]\mathrm{d}u_1\mathrm{d}u_2-\frac{\mu^2}{\sigma^2}, \tag{2}$$

where $F^{-1}(\cdot)$ is the inverse CDF of the non-Gaussian field; $c_{\mathrm{G}}(u_1,u_2;\rho_{\mathrm{G}})$ represents the standard bivariate Gaussian copula density function with parameter ρ_{G}. Eq. (2) establishes a unique, monotonic mapping between $\rho(\boldsymbol{\xi})$ and $\rho_{\mathrm{G}}(\boldsymbol{\xi})$ at each spatial lag $\boldsymbol{\xi}$.

In engineering practice where observational data of the random field are available, the transformed correlation function can be directly estimated in a data-driven manner. To eliminate potential errors arising from parametric marginal distribution assumptions, the marginal CDF $F(y)$ can be substituted with the empirical CDF. The Gaussianized data points $z^{(i)}$ corresponding to observations $y^{(i)}$ at discrete spatial locations $\boldsymbol{x}^{(i)}$, for $i=1,\ldots,n$, are obtained via

$$z^{(i)}=\Phi^{-1}\left[\frac{1}{n}\sum_{j=1}^{n}u\left(y^{(i)}-y^{(j)}\right)\right], \tag{3}$$

where $u(\cdot)$ is the Heaviside step function. Common correlation kernels can then be directly fitted to the auxiliary data $\{z^{(i)}\}$ to construct $\rho_{\mathrm{G}}(\boldsymbol{\xi})$, enabling consistent modeling and straightforward non-iterative simulation of univariate non-Gaussian fields.

3. Multivariate Modeling via Extended Bridge Function

To describe a bivariate homogeneous non-Gaussian random field, denoted as $\boldsymbol{Y}(\boldsymbol{x})=(Y_1(\boldsymbol{x}),Y_2(\boldsymbol{x}))^{\mathrm{T}}$, three distinct aspects of probabilistic uncertainty must be modeled: (1) the individual marginal distributions of the variables, (2) the spatial correlation of each individual random field, and (3) the pointwise cross-variable dependence between the two fields. Let the marginal distributions of $Y_1(\boldsymbol{x})$ and $Y_2(\boldsymbol{x})$ be represented by their respective CDFs, $F_1(y_1)$ and $F_2(y_2)$. Following the framework established in Section 2, the spatial variability of each field is characterized by their transformed correlation functions, $\rho_{1\mathrm{G}}(\boldsymbol{\xi})$ and $\rho_{2\mathrm{G}}(\boldsymbol{\xi})$, which correspond to the auto-correlation functions of the marginally Gaussianized auxiliary fields $Z_1(\boldsymbol{x})=\Phi^{-1}\{F_1[Y_1(\boldsymbol{x})]\}$ and $Z_2(\boldsymbol{x})=\Phi^{-1}\{F_2[Y_2(\boldsymbol{x})]\}$. Finally, the pointwise probabilistic dependence between $Y_1(\boldsymbol{x})$ and $Y_2(\boldsymbol{x})$ at any spatial location $\boldsymbol{x}$ is described using copula theory (Tao and Chen, 2023). According to Sklar's theorem, the joint CDF of the bivariate variables can be represented as $F(y_1,y_2)=C_{12}[F_1(y_1),F_2(y_2)]$, where $C_{12}(\cdot,\cdot)$ is the copula function.

To simulate these coupled fields while preserving both their unique spatial correlations and their non-Gaussian cross-dependence, a transformation mechanism is required to link the variables. Based on the conditional copula method, the realization of the second field $Y_2(\boldsymbol{x})$ can be mathematically expressed as a function of the first field $Y_1(\boldsymbol{x})$ and an auxiliary random field $Z_{\mathrm{b}}(\boldsymbol{x})$ (Liu et al., 2025; Lyu et al., 2025), namely,

$$Y_2(\boldsymbol{x})=F_2^{-1}\left(C_{2|1}^{-1}\{\Phi[Z_{\mathrm{b}}(\boldsymbol{x})]\,|F_1[Y_1(\boldsymbol{x})]\,\}\right), \tag{4}$$

where $C_{2|1}^{-1}(\cdot|\cdot)$ represents the inverse of the conditional copula function $C_{2|1}(u_2|u_1)=\partial C_{12}(u_1,u_2)/\partial u_1$; $Z_{\mathrm{b}}(\boldsymbol{x})$ is a standard homogeneous Gaussian field that is statistically independent of $Y_1(\boldsymbol{x})$. The field $Z_{\mathrm{b}}(\boldsymbol{x})$ is defined as the *bridge field*, and its spatial auto-correlation function is denoted as $\rho_{\mathrm{b}}(\boldsymbol{\xi})=\mathrm{E}[Z_{\mathrm{b}}(\boldsymbol{x})Z_{\mathrm{b}}(\boldsymbol{x}+\boldsymbol{\xi})]$. The bridge field acts as an auxiliary spatial carrier; its correlation structure $\rho_{\mathrm{b}}(\boldsymbol{\xi})$ governs how the spatial variability of the first field is mapped and transmitted to the second field.

Since Eq. (4) establishes a deterministic point-wise relationship among $Y_1(\boldsymbol{x})$, $Y_2(\boldsymbol{x})$, and $Z_{\mathrm{b}}(\boldsymbol{x})$, there must exist a corresponding functional relationship among their spatial correlation properties. Specifically, the transformed correlation function of the second field, $\rho_{2\mathrm{G}}(\boldsymbol{\xi})$, can be derived as a function of the first field's transformed correlation $\rho_{1\mathrm{G}}(\boldsymbol{\xi})$ and the bridge field's correlation $\rho_{\mathrm{b}}(\boldsymbol{\xi})$. By definition, the transformed correlation $\rho_{2\mathrm{G}}(\boldsymbol{\xi})$ is the Pearson correlation of the Gaussianized variables $Z_2(\boldsymbol{x})=\Phi^{-1}\{F_2[Y_2(\boldsymbol{x})]\}$ and $Z_2(\boldsymbol{x}+\boldsymbol{\xi})=\Phi^{-1}\{F_2[Y_2(\boldsymbol{x}+\boldsymbol{\xi})]\}$. Substituting the conditional copula mapping into this definition yields

$$\rho_{2\mathrm{G}}(\boldsymbol{\xi})=\mathrm{E}\left[\Phi^{-1}\left(C_{2|1}^{-1}\{\Phi[Z_{\mathrm{b}}(\boldsymbol{x})]\,|F_1[Y_1(\boldsymbol{x})]\,\}\right)\Phi^{-1}\left(C_{2|1}^{-1}\{\Phi[Z_{\mathrm{b}}(\boldsymbol{x}+\boldsymbol{\xi})]\,|F_1[Y_1(\boldsymbol{x}+\boldsymbol{\xi})]\,\}\right)\right]. \tag{5}$$

Because $Y_1(\boldsymbol{x})$ and $Z_{\mathrm{b}}(\boldsymbol{x})$ are mutually independent fields, their joint PDF factorizes. By converting the expectation into an integral over the unit hypercube $[0,1]^4$ using probability integral transforms, the relationship can be written as

$$\begin{aligned}\rho_{2\mathrm{G}}(\boldsymbol{\xi})&=r_{\mathrm{b}}[\rho_{\mathrm{b}}(\boldsymbol{\xi})\,|\rho_{1\mathrm{G}}(\boldsymbol{\xi})\,]\\&=\int_{[0,1]^4}\Phi^{-1}[C_{2|1}^{-1}(u_{\mathrm{b}}|u_1)]\Phi^{-1}[C_{2|1}^{-1}(u_{\mathrm{b}}'|u_1')]c_G[u_{\mathrm{b}},u_{\mathrm{b}}';\rho_{\mathrm{b}}(\boldsymbol{\xi})]c_G[u_1,u_1';\rho_{1\mathrm{G}}(\boldsymbol{\xi})]\mathrm{d}u_1\mathrm{d}u_{\mathrm{b}}\mathrm{d}u_1'\mathrm{d}u_{\mathrm{b}}',\end{aligned} \tag{6}$$

where $c_{\mathrm{G}}(\cdot,\cdot;\rho)$ denotes the bivariate standard Gaussian copula density function with correlation parameter ρ. The functional operator $r_{\mathrm{b}}(\cdot|\cdot)$ is defined as the *bridge function* of the bivariate non-Gaussian field.

A fundamental property of the extended bridge function derived in Eq. (6) is its *marginal distribution invariance*. Although the physical fields $Y_1(\boldsymbol{x})$ and $Y_2(\boldsymbol{x})$ may possess highly non-Gaussian, skewed, or bounded marginal distributions (such as Gamma, Lognormal, or Weibull distributions), the final integral in Eq. (6) depends exclusively on the conditional copula $C_{2|1}$ and the standard normal distribution functions. This invariance occurs because copulas are unaffected by strictly monotonic transformations of the marginal variables. Consequently, the bridge function is a purely copula-dependent operator. This property provides significant practical utility: the spatial mapping relation does not need to be re-derived or modified when the marginal distributions of the engineering parameters change, as long as the underlying copula structure representing their joint dependence remains consistent.

For a given target copula and specified transformed correlations $\rho_{1\mathrm{G}}(\boldsymbol{\xi})$ and $\rho_{2\mathrm{G}}(\boldsymbol{\xi})$, the only unknown parameter in Eq. (6) is the correlation function of the bridge field, $\rho_{\mathrm{b}}(\boldsymbol{\xi})$. Because the bridge function $r_{\mathrm{b}}(\rho_{\mathrm{b}}|\rho_{1\mathrm{G}})$ is strictly monotonic and bounded with respect to $\rho_{\mathrm{b}}\in[-1,1]$, the mapping is invertible. Thus, the required bridge correlation function can be uniquely determined at each spatial lag $\boldsymbol{\xi}$ as

$$\rho_{\mathrm{b}}(\boldsymbol{\xi})=r_{\mathrm{b}}^{-1}[\rho_{2\mathrm{G}}(\boldsymbol{\xi})\,|\rho_{1\mathrm{G}}(\boldsymbol{\xi})\,]. \tag{7}$$

In practical computations, rather than performing the four-dimensional integration in Eq. (6) at every spatial coordinate, the bridge function can be pre-computed numerically over a discrete grid of ρ_{b} and $\rho_{1\mathrm{G}}$

to construct a high-precision look-up table. A fast, one-dimensional interpolation can then be utilized to solve Eq. (7), ensuring excellent computational efficiency for subsequent stochastic simulations.

4. Non-Iterative Simulation Algorithms

Traditional simulation approaches for non-Gaussian random fields, such as translation field methods or Hermite polynomial models, often rely on iterative adjustments to correct correlation distortions caused by marginal transformations (Zhang et al. 2019, Zheng et al., 2023). These iterative corrections are computationally intensive and can suffer from convergence issues, especially during large-scale Monte Carlo trials. In contrast, by directly employing the transformed correlation functions ($\rho_{1\mathrm{G}}$, $\rho_{2\mathrm{G}}$) and the bridge correlation (ρ_{b}) established in Sections 2 and 3, the proposed framework enables a fully non-iterative simulation workflow. The computationally demanding step of mapping the correlation functions is decoupled from the field generation process, turning the simulation into a direct, feed-forward sequence of standard Gaussian simulation and point-wise transformations.

For a bivariate homogeneous non-Gaussian random field $(Y_1(\boldsymbol{x}), Y_2(\boldsymbol{x}))^{\mathrm{T}}$, the simulation procedure is executed through the following structured steps:

Step 1: Correlation preparation. Given the target transformed correlation functions $\rho_{1\mathrm{G}}(\boldsymbol{\xi})$ and $\rho_{2\mathrm{G}}(\boldsymbol{\xi})$ alongside the selected copula function $C_{12}(u_1, u_2)$, construct the offline bridge function $r_{\mathrm{b}}(\cdot|\cdot)$ via Eq. (6). Determine the required correlation function $\rho_{\mathrm{b}}(\boldsymbol{\xi})$ of the auxiliary bridge field $Z_{\mathrm{b}}(\boldsymbol{x})$ by numerically inverting Eq. (7) using a pre-computed lookup table and one-dimensional interpolation.

Step 2: Auxiliary Gaussian field generation. Generate two mutually independent, homogeneous, standard Gaussian random fields: $Z_1(\boldsymbol{x})$ with the auto-correlation function $\rho_{1\mathrm{G}}(\boldsymbol{\xi})$, and $Z_{\mathrm{b}}(\boldsymbol{x})$ with the auto-correlation function $\rho_{\mathrm{b}}(\boldsymbol{\xi})$. This step can be performed using any classical, highly efficient Gaussian simulation technique, such as the spectral representation method (Deodatis and Shields, 2024) or the stochastic harmonic function (SHF) method (Fu et al., 2026).

Step 3: Generation of the first field. Obtain the realization of the first non-Gaussian random field $Y_1(\boldsymbol{x})$ by applying the inverse standard normal transformation to $Z_1(\boldsymbol{x})$: $Y_1(\boldsymbol{x}) = F_1^{-1}\{\Phi[Z_1(\boldsymbol{x})]\}$, where $F_1(\cdot)$ is the target marginal CDF of $Y_1(\boldsymbol{x})$.

Step 4: Generation of the second field. Using the generated Gaussianized auxiliary realization $Z_1(\boldsymbol{x})$ (where $\Phi[Z_1(\boldsymbol{x})] = F_1[Y_1(\boldsymbol{x})]$) and the auxiliary bridge field $Z_{\mathrm{b}}(\boldsymbol{x})$, compute the realization of the second non-Gaussian random field $Y_2(\boldsymbol{x})$ point-wise via the inverse conditional copula mapping: $Y_2(\boldsymbol{x}) = F_2^{-1}(C_{2|1}^{-1}\{\Phi[Z_{\mathrm{b}}(\boldsymbol{x})] | \Phi[Z_1(\boldsymbol{x})]\})$, where $F_2(\cdot)$ is the target marginal CDF of $Y_2(\boldsymbol{x})$.

The primary advantage of this workflow is its numerical robustness and speed. Because the mapping of the bridge correlation $\rho_{\mathrm{b}}(\boldsymbol{\xi})$ is solved prior to sample generation, the runtime of generating each bivariate realization is practically identical to that of generating two independent Gaussian random fields plus a few elementary point-wise operations. This direct nature makes the algorithm highly attractive for reliability analysis and stochastic finite element applications where thousands of realizations are required. Furthermore, this bivariate workflow serves as the foundational building block for higher-dimensional fields, where vine copulas can be decomposed into hierarchical sequences of these bivariate steps.

5. Numerical Case Study

To demonstrate the application of the proposed transformation-based framework in engineering computations, a numerical case study involving the modeling and simulation of two-dimensional (2D) concrete material properties is presented. In the phase-field theory of concrete cracking, tensile strength $f_{\mathrm{t}}(\boldsymbol{x})$ and fracture energy $G_{\mathrm{c}}(\boldsymbol{x})$ are two vital parameters that exhibit significant spatial variability and physical coupling (Hai and Lyu, 2023). In this study, the tensile strength field $Y_1(\boldsymbol{x})=f_{\mathrm{t}}(\boldsymbol{x})$ is assumed to follow a Gaussian distribution with a mean $\mu_f=3.5\times10^6$ Pa and a standard deviation $\sigma_f=8.5\times10^5$ Pa. The fracture energy field $Y_2(\boldsymbol{x})=G_{\mathrm{c}}(\boldsymbol{x})$ is modeled as a lognormal distribution with mean $\mu_G=4.6\ \mathrm{J/m^2}$ and standard deviation $\sigma_G=0.40\ \mathrm{J/m^2}$. Both properties are modeled over an isotropic, homogeneous 2D spatial domain of 0.2 m × 0.2 m. Their spatial auto-correlation functions share the same squared exponential form as

$$\rho(\xi_1,\xi_2)=e^{-\frac{\pi(\xi_1{}^2+\xi_2{}^2)}{l^2}}, \tag{8}$$

where ξ_1 and ξ_2 represent the spatial distances in the horizontal and vertical directions, and the correlation length is specified as $l=0.05\ \mathrm{m}$.

The point-wise probabilistic dependency between the tensile strength and fracture energy is modeled using a Student's t-copula (Hai and Lyu, 2023). This copula structure allows the representation of symmetric tail dependencies that are often observed in experimental material testing. The joint copula is specified as

$$C_{fG}(u_1,u_2)=T^{(2)}[T^{-1}(u_1;\nu_{\mathrm{t}}),T^{-1}(u_2;\nu_{\mathrm{t}});\rho_{\mathrm{t}},\nu_{\mathrm{t}}], \tag{9}$$

where $T(\cdot;\nu_{\mathrm{t}})$ is the CDF of the standard univariate Student's t-distribution with ν_{t} degrees of freedom; $T^{(2)}(\cdot,\cdot;\rho_{\mathrm{t}},\nu_{\mathrm{t}})$ is the bivariate Student's t-CDF; $u_1=F_1(f_{\mathrm{t}})$, and $u_2=F_2(G_{\mathrm{c}})$. The model parameters are set to $\rho_{\mathrm{t}}=-0.14$ and $\nu_{\mathrm{t}}=5.0$. Because $f_{\mathrm{t}}(\boldsymbol{x})$ is Gaussian, its transformed correlation function $\rho_{1\mathrm{G}}$ is identical to its original correlation ρ. For the lognormal fracture energy $G_{\mathrm{c}}(\boldsymbol{x})$, the mapping in Eq. (2) is employed to numerically solve for its transformed correlation function $\rho_{2\mathrm{G}}$. Using the bivariate copula parameters, the bridge function $r_{\mathrm{b}}(\rho_{\mathrm{b}}|\rho_{1\mathrm{G}})$ is pre-computed, and the auto-correlation function $\rho_{\mathrm{b}}(\boldsymbol{\xi})$ of the auxiliary bridge field $Z_{\mathrm{b}}(\boldsymbol{x})$ is subsequently determined through the inverse mapping in Eq. (7).

Realizations of the coupled 2D fields are generated on a discretized 100 × 100 spatial grid using the non-iterative simulation algorithm detailed in Section 4. The independent auxiliary Gaussian fields $Z_1(\boldsymbol{x})$ and $Z_{\mathrm{b}}(\boldsymbol{x})$ are simulated using the spectral representation method (Deodatis and Shields, 2024). A representative pair of generated sample fields is visualized in Figure 1. Figure 1 (a) displays the spatial distribution of the simulated tensile strength $f_{\mathrm{t}}(\boldsymbol{x})$, while Figure 1 (b) illustrates the corresponding fracture energy field $G_{\mathrm{c}}(\boldsymbol{x})$. The spatial patterns in both plots visually reflect the isotropic correlation structure and the prescribed correlation length of 0.05 m, demonstrating that the spatial variations are captured smoothly.

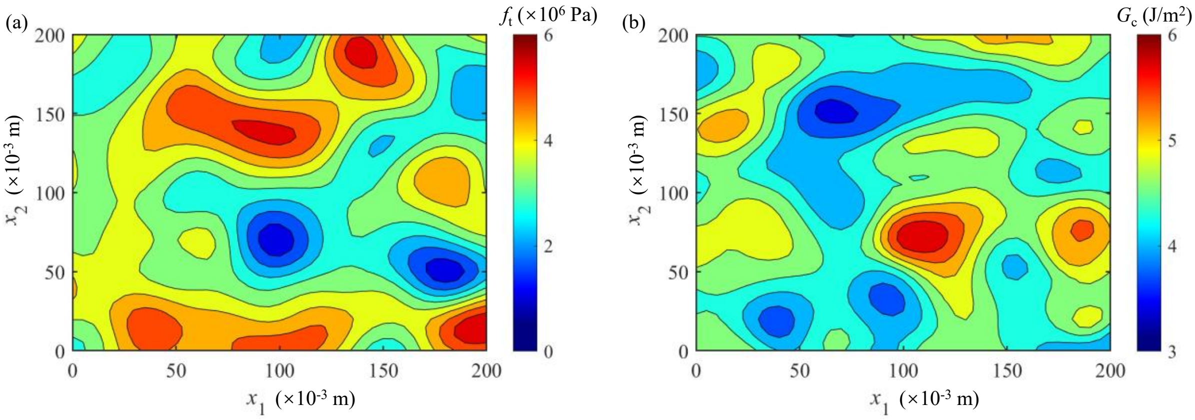

Figure 1. Representative simulated realizations of the 2D random fields over the 0.2 m × 0.2 m domain: (a) Tensile strength; (b) Fracture energy.

To quantitatively verify the accuracy and consistency of the simulation method, a statistical analysis is conducted over a set of 10^4 independent realizations. Figure 2 summarizes the validation of the spatial auto-correlations and the point-wise joint PDF. In Figure 2 (a), the auto-correlation functions computed from the simulated samples are compared against their respective analytical targets for both $f_{\mathrm{t}}(\boldsymbol{x})$ and $G_{\mathrm{c}}(\boldsymbol{x})$. The simulated auto-correlations match the target curves closely, confirming that the transformation-based correlation mapping successfully preserves the spatial structure without requiring any iterative correction steps. Furthermore, to evaluate the preservation of the non-Gaussian joint dependency, Figure 2 (b) overlays the scatter plot of the simulated sample points (sampled at a specific spatial location across the ensemble) onto the theoretical joint PDF contours of the Student's t-copula. The simulated points conform closely to the shape of the joint PDF contours, showcasing the capability of the extended bridge function framework to simultaneously reproduce the spatial correlation of individual fields and their non-Gaussian, non-linear mutual dependence.

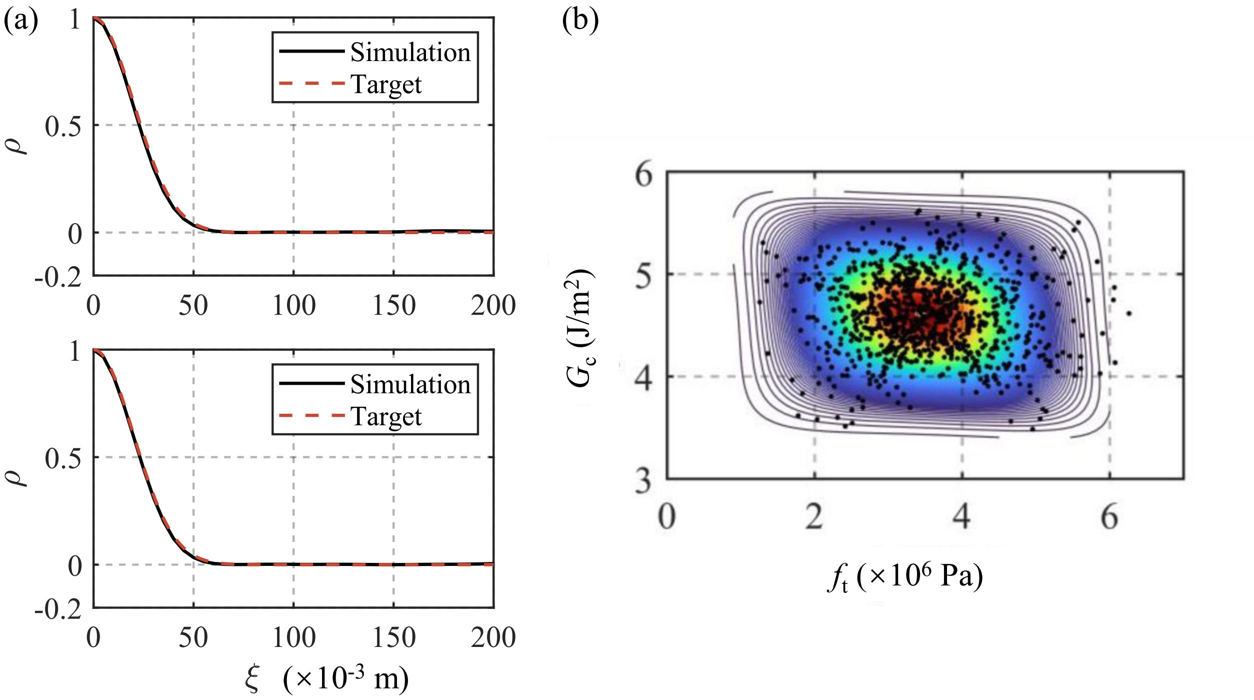

Figure 2. Statistical validation of the simulated random fields: (a) Comparison between the target and simulated auto-correlation functions; (b) Scatter plot of simulated realizations overlaid on the theoretical joint PDF contours of tensile strength and fracture energy.

6. Concluding Remarks

In this study, a transformation-based framework has been developed for the probabilistic modeling and simulation of bivariate non-Gaussian random fields. By introducing the transformed correlation function, the proposed approach represents spatial variability directly through the linear correlation of standard Gaussianized auxiliary variables, which eliminates the modeling incompatibility found in classical translation field methods. Additionally, the bridge function concept is extended to describe the mapping between individual spatial correlations and pointwise non-Gaussian dependencies. This extended bridge function is invariant to marginal transformations, remaining solely dependent on the underlying copula structure.

The corresponding simulation algorithm is direct and non-iterative, offering a computationally attractive option for uncertainty quantification where large-scale realizations are needed. Numerical validation using a coupled concrete property case study demonstrates that the generated fields conform closely to both the target marginal distributions, the individual auto-correlation functions, and the prescribed joint probability density.

Future research may focus on extending this framework to non-stationary random fields, exploring non-Gaussian copula dependencies across different spatial lags, and applying the proposed simulation method to large-scale structural reliability and stochastic finite element analyses in safety-critical engineering systems.

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