A linear-moment-based sensitivity analysis method considering the influence of correlation coefficient [*]

Qinghe Li, Xuan-Yi Zhang[†], and Yan-Gang Zhao
National Key Laboratory of Bridge Safety and Resilience, Beijing University of Technology, Beijing 100124, China liqinghe@emails.bjut.edu.cn
National Key Laboratory of Bridge Safety and Resilience, Beijing University of Technology, Beijing 100124, China zhangxuanyi@bjut.edu.cn
National Key Laboratory of Bridge Safety and Resilience, Beijing University of Technology, Beijing 100124, China zhaoyg@bjut.edu.cn

Abstract. A reliability sensitivity analysis method based on linear moments (L-moments) is proposed in this paper to investigate the sensitivity of the failure probability to the correlation coefficients between input random variables. The sensitivity index is defined as the partial derivative of the failure probability with respect to the correlation coefficients that define the dependence structure among the inputs. A complete expression for the correlation sensitivity index is derived using the first four L-moments. As a by-product of the L-moments-based reliability analysis, the proposed method efficiently evaluates sensitivity information without requiring additional costly structural analyses. The effectiveness of the proposed method is demonstrated through a numerical example. The results show excellent agreement with those obtained from Monte Carlo simulation confirming the accuracy of the method.

Keywords: Sensitivity analysis, Linear moments, Correlation coefficient

1. Introduction

During daily service, structures will face two aspects of uncertainty: one is the uncertainty of external loads, and the other is the uncertainty of their own structure parameters(Zhang et al., 2021; Guo et al., 2023; Zhao et al., 2022). In order to evaluate these uncertainties, structural reliability theory has been widely applied, and commonly used reliability theories include first-order moment method(FORM)(Lu et al., 2017), second-order moment method(SORM)(Zhao et al., 2002),simulation method(SS)(Au and Beck, 2001), probability density evolution method(PDEM)(Li, 2016), MOM(Zhao and Ono, 2001), etc, the development of reliability theory is relatively mature. However, the impact of random input variables on failure probability, namely sensitivity analysis, is relatively limited. In order to provide assistance for structural optimization design, sensitivity analysis methods are important. Meanwhile, in engineering practice, structural parameters are often not independent of each other and have strong correlations. And then, there is currently

[*] Footnote to the title with the 'thanks' command.
[†] Corresponding author

limited research on sensitivity analysis, and there is no sensitivity analysis method specifically for correlation coefficients. Therefore, it is necessary to study the sensitivity methods for correlation coefficients.

Correlation coefficient is a measure that describes the correlation between parameters, ignoring the influence of correlation can lead to significant errors, and the failure probability also shows a high sensitivity to correlation. Previous research on sensitivity mainly focused on the random variables themselves and their distribution, giving rise to many methods, such as single-loop estimation method(Wei et al., 2012), maximum entropy-based method(Yun et al., 2019a), Bayes formula-based method(Wang et al., 2018; Wang et al., 2019; Yun et al., 2019b), and sensitivity analysis based on methods of moments (SAMOM)(Zhang et al., 2025), etc. However, these methods all ignore correlation, the failure probability also shows a high sensitivity to the correlation coefficient.

In this paper, the sensitivity of calculating the correlation between parameters is actually a local sensitivity analysis, which involves calculating the partial derivative of the failure probability with respect to the correlation coefficient. However, to calculate the partial derivative of the failure probability with respect to the correlation coefficient, the first step is to calculate the failure probability. Sensitivity analysis methods are mostly designed as post-processing for reliability analysis. Both FORM based and SORM based reliability methods require some assumptions, which result in poor accuracy when dealing with strongly nonlinear problems. Although the method of moment based on central moments(C-moments) can handle strongly nonlinear problems, it requires a huge sample size. The method of moment based on linear moments(L-moments)(Hosking, 2018) can handle strongly nonlinear problems while requiring a smaller sample size and also has good performance for high-dimensional problems.

This article proposes a sensitivity analysis method for correlation coefficients based on linear moments, which calculates the partial derivative of the failure probability with respect to the correlation coefficient. This method has the following improvements, it is a byproduct of reliability analysis. It clarifies the influence of correlation coefficients on failure probability and can effectively handle strong nonlinear problems.

2. PROBLEM STATEMENT

2.1. Failure Probability

In structural reliability analysis, the performance functions are typically used to describe the working state of a structure or system under random input. The performance function is represented as $G(\mathrm{X})$, where $\mathrm{X} = (x_1, x_2, ..., x_3)^T$ represents a vector of n random variables reflecting uncertainties in model parameters and external loading conditions. When $G(\mathrm{X}) \leq 0$, it indicates structural failure, and when $G(\mathrm{X}) > 0$, it indicates that the structure is safe. Based on the definition, the failure probability $P_F(\boldsymbol{\rho})$ is defined by a multi-fold integral as follows:

$$P_F(\boldsymbol{\rho}) = \int\limits_{G(\mathrm{X})\leq 0} f_{\mathrm{X}}(\mathrm{X} \mid \boldsymbol{\rho}) dx \tag{1}$$

where $P_F(\boldsymbol{\rho})$ is the failure probability; $f_{\mathrm{X}}(\mathrm{X} \mid \boldsymbol{\rho})$ is the joint probability density function (JPDF) of X; $\boldsymbol{\rho}$ is a vector that collects the correlation coefficient between random variables, and $\boldsymbol{\rho} = (\rho_{i1}, \rho_{i2}, ..., \rho_{ij}, ..., \rho_{in})$, ρ_{ij} is the correlation coefficient between the $i-th$ random variable and the $j-th$ random variable, $\rho_{ij} = \rho_{ji}$; With $P_F(\boldsymbol{\rho})$ is determined, the reliability index $\beta(\boldsymbol{\rho})$ can be obtained as follows:

$$\beta(\boldsymbol{\rho}) = -\Phi^{-1}\left[P_F(\boldsymbol{\rho})\right] \tag{2}$$

where $\Phi^{-1}[\cdot]$ is the inverse cumulative distribution function(CDF) of the standard normal distribution.

2.2. Sensitivity index of failure probability

The failure probability defined in Eq.(1) is controlled by many important factors, one of which is the correlation between input random variables. Therefore, it is important to understand the impact of correlation coefficient between parameters on the probability of failure. The sensitivity can be quantified through the partial derivative of $P_F(\boldsymbol{\rho})$ with respect to correlation coefficient ρ_{ij}. In order to be applicable for comparison correlation coefficient between different random variables, a dimensionless measure is provided, which uses an elasticity based sensitivity index, which is denoted by $\varepsilon_{\theta_{ij}}$ and defined as follows(Zhang et al., 2025):

$$\epsilon_{\boldsymbol{\rho}} = \frac{\partial P_F(\boldsymbol{\rho})}{\partial \rho_{ij}} \cdot \frac{\rho_{ij}}{P_F(\boldsymbol{\rho})} \tag{3}$$

The partial derivative of $P_F(\boldsymbol{\rho})$ with respect to ρ_{ij} can be computed as shown in Equation.4(Wu, 1993).

$$\frac{\partial P_F(\boldsymbol{\rho})}{\partial \rho_{ij}} = \int\limits_{G(\mathbf{X})\leq 0} \frac{\partial [f_{\mathrm{X}}(\mathrm{X}|\boldsymbol{\rho})]}{\partial \rho_{ij}} d\mathbf{x} \tag{4}$$

The joint probability density function(JPDF) between input random variables can be expressed as:

$$f_{\mathrm{X}}(\boldsymbol{x} \mid \boldsymbol{\rho}) = c(z_1, z_2, \ldots, z_n) \prod_{i=1}^{n} f_{X_i}(x_i) \tag{5}$$

where, ρ is the correlation coefficient matrix; $c(z_1, z_2, ..., z_n)$ is the copula function. z is a column vector, which is the inverse cumulative distribution function of the random variable, $z = z_1, z_2, ..., z_n$, $z_i = F_i(x_i)$, x_i is the random variable, i is the number of random variables. $f_{\mathrm{X}}(\boldsymbol{x} \mid \boldsymbol{\rho})$ is the the joint probability density function of random variables. $f(x_i)$ is the probability density function of x_i. And then, the partial derivative of the failure probability $P_F(\boldsymbol{\rho})$ on the correlation coefficient ρ_{ij} can be expressed as:

$$\frac{\partial P_F(\boldsymbol{\rho})}{\partial \rho_{ij}} = \int\limits_{G(\mathbf{X})\leq 0} h_{ij}(x_i \mid \boldsymbol{\theta_i}) f_{\mathrm{X}}(\mathrm{X} \mid \boldsymbol{\theta}) dx \tag{6}$$

$$h_{ij}(x_i \mid \rho_{ij}) = \frac{\partial c(z_1, z_2, \ldots, z_n)}{\partial \rho_{ij}} \cdot \frac{1}{c(z_1, z_2, \ldots, z_n)} \tag{7}$$

However, it's difficult to directly evaluate Eqs.(1) and (4), because of the performance function usually strongly nonlinear and implicit. In addition, the failure domain is difficult to determine in high-dimensional structural engineering problems. In order to solve the problem, an efficient computational approach based on the method of L-moments is adopted.

3. REVIEW OF THE METHOD OF LINEAR MOMENTS

In the reliability analysis based on the method of L-moments, the performance function $G(\mathrm{X})$ can be regarded as a random variable Y, consequently, $P_F(\boldsymbol{\rho})$ can be defined by evaluating the cumulative distribution function(CDF) of Y at the limit state $Y = 0$, shown in the following equation(Zhao and Ono, 2001):

$$P_F(\boldsymbol{\rho}) = F_Y\left[0 \mid \lambda_G\right] \tag{8}$$

where, $F_Y[0 \mid \boldsymbol{\lambda_G}]$ represents the CDF of $Y = G(X)$; $\boldsymbol{\lambda}_G = (\lambda_{G1}, \lambda_{G2}, \lambda_{G3}, \lambda_{G4})^T$ denotes the vector of the first L-moments of Y, among them, λ_{G1}, λ_{G2}, λ_{G3}, λ_{G4} are the L-mean, L-standard deviation, L-skewness and L-kurtosis of Y, respectively. In practical engineering applications, the exact distribution of Y is typically unknown and it is difficult to theoretical construct $F_Y[0 \mid \boldsymbol{\lambda_G}]$. By approximating non normal random variables with a cubic equation of standard normal random variables, a normal transformation ($U - Y$ transformation) can be applied(Zhao et al., 2020):

$$Y = a + bU + cU^2 + dU^3 \tag{9}$$

where Y is the non-normal random variable, U is the standard normal random variable; a, b, c, d are the polynomial coefficient, which can be explicitly defined in terms of λ_G(Tung, 2000):

$$a = \lambda_{G1} - 1.81379937\lambda_{G3} \tag{10}$$

$$b = 2.25518617\lambda_{G2} - 3.93740250\lambda_{G4} \tag{11}$$

$$c = 1.81379937\lambda_{G3} \tag{12}$$

$$d = -0.19309293\lambda_{G2} + 1.574961\lambda_{G4} \tag{13}$$

Based on Eq.(9), $P_F(\boldsymbol{\rho})$ can be obtained as follows:

$$P_F(\boldsymbol{\rho}) = \Phi[-\beta_M(\boldsymbol{\lambda}_G)] \tag{14}$$

where $\beta_M(\boldsymbol{\lambda}_G)$ is the reliability index based on linear moments, generally, $\beta_M(\boldsymbol{\lambda}_G)$ can be expressed as Eq.(15)(Zhao et al., 2020; Tong et al., 2021; Tong et al., 2025), for other situations, please refer to the Appendix A.

$$\beta_M(\boldsymbol{\lambda}_G) = -\sqrt[3]{-\frac{h}{2} - \sqrt{A}} - \sqrt[3]{-\frac{h}{2} + \sqrt{A}} + \frac{c}{3d} \tag{15}$$

where parameters A and h can be expressed as:

$$A = \left(-\frac{c^2 - 3bd}{9d^2}\right)^3 + \frac{h^2}{4} \tag{16}$$

$$h = \frac{2c^3}{27d^3} - \frac{bc}{3d^2} + \frac{a}{d} \tag{17}$$

The first four linear moments λ_G are obtained based on the probability-weighted moments(PWMs) $\mathbf{M} = (M_0, M_1, M_2, M_3)^T$ are expressed as:

$$\lambda_{G1} = M_0 \tag{18}$$

$$\lambda_{G2} = 2M_1 - M_0 \tag{19}$$

$$\lambda_{G3} = 6M_2 - 6M_1 + M_0 \tag{20}$$

$$\lambda_{G4} = 20M_3 - 30M_2 + 12M_1 - M_0 \tag{21}$$

The expression for the probability weight moment: is

$$M_r = \int_0^1 y \cdot [F_Y(y|\boldsymbol{\rho})]^r \, dF_Y(y|\boldsymbol{\rho}) = \int_{\Omega_X} y \cdot [F_Y(y|\boldsymbol{\rho})]^r \cdot f_X(x|\boldsymbol{\rho}) dx, \quad (r = 0, 1, 2, 3) \tag{22}$$

where, $F_Y(y|\boldsymbol{\rho})$ is the CDF of Y, which is defined as:

$$F_Y(y|\boldsymbol{\rho}) = \int_{\Omega_X} I[Y \leq y] \cdot f_X(\mathbf{x}|\boldsymbol{\rho}) \, d\mathbf{x} \tag{23}$$

and $I[Y \leq y]$ denotes the indicator function.

4. Reliability sensitivity analysis method based on linear moments

Based on Eqs.(14)-(23) and the differential chain rule, the partial derivative of failure probability $P_F(\boldsymbol{\rho})$ with respect to correlation coefficient ρ_{ij} can be derived and expressed as:

$$\frac{\partial P_F(\boldsymbol{\rho})}{\partial \rho_{ij}} = \frac{\partial P_F}{\partial \beta_M} \cdot \frac{\partial \beta_M}{\partial \boldsymbol{\lambda_G}} \cdot \frac{\partial \boldsymbol{\lambda_G}}{\partial \mathbf{M}_G} \cdot \frac{\partial \mathbf{M}_G}{\partial \rho_{ij}} \tag{24}$$

each of the four terms on the right-hand side of Eq.(24) is defined separately.

For the first item of the Eq.(24), the derivative of $P_F(\boldsymbol{\rho})$ with respect to the L-moments-based reliability index $\beta_M(\boldsymbol{\lambda}_G)$ is obtained as follows Eq.(25) based on Eq.(14).

$$\frac{\partial P_F(\boldsymbol{\rho})}{\partial \beta_M(\boldsymbol{\lambda}_G)} = -\varphi\left[-\beta_M(\boldsymbol{\lambda}_G)\right] \tag{25}$$

where $\varphi(\boldsymbol{\lambda}_G)$ represents the PDF of the standard normal distribution.

For the second item of the Eq.(24), $\beta_M(\boldsymbol{\lambda}_G)$ is defined based on the first four L-moments of the performance function, so the derivative of $\beta_M(\boldsymbol{\lambda}_G)$ with respect to $\boldsymbol{\lambda}_G$ is a vector composed of four elements, which can be expressed as:

$$\frac{\partial \beta_M(\boldsymbol{\lambda}_G)}{\partial \boldsymbol{\lambda}_G} = \left(\frac{\partial \beta_M(\boldsymbol{\lambda}_G)}{\partial \lambda_{G1}}, \frac{\partial \beta_M(\boldsymbol{\lambda}_G)}{\partial \lambda_{G2}}, \frac{\partial \beta_M(\boldsymbol{\lambda}_G)}{\partial \lambda_{G3}}, \frac{\partial \beta_M(\boldsymbol{\lambda}_G)}{\partial \lambda_{G4}} \right) \tag{26}$$

The expression for $\partial\beta_M(\boldsymbol{\lambda}_G)/\partial\lambda_{Gi}$ can be obtained by performing the central finite difference method on formula Eq.(15), as shown in the following equation

$$\frac{\partial\beta_M(\boldsymbol{\lambda}_G)}{\partial\lambda_{Gi}} = \frac{\beta_M(\boldsymbol{\lambda}_G \mid \lambda_{Gi} + 1e^{-5}) - \beta_M(\boldsymbol{\lambda}_G \mid \lambda_{Gi} - 1e^{-5})}{2e^{-5}} \tag{27}$$

As shown in Appendix A, the types of $\beta_M(\boldsymbol{\lambda}_G)$ are different depending on the different linear moments ratios, the solutions for other situations can also be obtained by differencing.

For the third item of the Eq.(24), $\partial\boldsymbol{\lambda}_G/\partial\mathbf{M}$ is a 4×4 matrix obtained based on the relationship between L-moments and PWMs as shown in Eq.(18)-Eq.(21), which can be obtained as follows:

$$\frac{\partial\boldsymbol{\lambda}_G}{\partial\mathbf{M}} = \begin{pmatrix} \frac{\partial\lambda_{G1}}{\partial M_0} & \frac{\partial\lambda_{G1}}{\partial M_1} & \frac{\partial\lambda_{G1}}{\partial M_2} & \frac{\partial\lambda_{G1}}{\partial M_3} \\ \frac{\partial\lambda_{G2}}{\partial M_0} & \frac{\partial\lambda_{G2}}{\partial M_1} & \frac{\partial\lambda_{G2}}{\partial M_2} & \frac{\partial\lambda_{G2}}{\partial M_3} \\ \frac{\partial\lambda_{G3}}{\partial M_0} & \frac{\partial\lambda_{G3}}{\partial M_1} & \frac{\partial\lambda_{G3}}{\partial M_2} & \frac{\partial\lambda_{G3}}{\partial M_3} \\ \frac{\partial\lambda_{G4}}{\partial M_0} & \frac{\partial\lambda_{G4}}{\partial M_1} & \frac{\partial\lambda_{G4}}{\partial M_2} & \frac{\partial\lambda_{G4}}{\partial M_3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & -6 & 6 & 0 \\ -1 & 12 & -30 & 20 \end{pmatrix} \tag{28}$$

For the fourth item of the Eq.(24), Based on Eq.(22), $\partial\mathbf{M}/\partial\rho_{ij}$ is divided into four elements, which can be expressed as shown in Eq.(29). Each term in this formula can be expressed as shown in Eq.(32).

$$\frac{\partial\mathbf{M}}{\partial\rho_{ij}} = \left(\frac{\partial M_0}{\partial\rho_{ij}}, \frac{\partial M_1}{\partial\rho_{ij}}, \frac{\partial M_2}{\partial\rho_{ij}}, \frac{\partial M_3}{\partial\rho_{ij}}\right)^T \tag{29}$$

Given N samples generated using the LHS method with a maximin distance criterion, the estimated PWMs and their derivatives can be approximated by:

$$\widehat{M_r} = \frac{1}{N}\sum_{l=1}^{N}\left\{y_{l:N} \cdot \left[\widehat{F}_Y(y_{l:N} \mid \boldsymbol{\rho})\right]^r\right\}, \quad (r = 0, 1, 2, 3) \tag{30}$$

where $y_{l:N}$ represents the $l-th$ order statistic of a sample of size N; The empirical CDF $\widehat{F_Y}(y_{l:N} \mid \boldsymbol{\rho})$ can be obtained as follows based on Eq.(23)

$$\widehat{F}_Y(y_{l:N} \mid \boldsymbol{\rho}) = \frac{1}{N}\sum_{l=1}^{N}\boldsymbol{I}[Y \leq y] = \frac{l}{N} \tag{31}$$

By using central finite differences, the expression of $\partial M_r/\partial\rho_{ij}$ can be obtained as follows:

$$\frac{\partial\widehat{M_r}}{\partial\rho_{ij}} = \frac{\sum_{l=1}^{N}\left\{y_{l:N} \cdot \left[\hat{F}_Y(y_{l:N} \mid \rho_{ij} + 1e^{-5})\right]\right\} - \sum_{l=1}^{N}\left\{y_{l:N} \cdot \left[\hat{F}_Y(y_{l:N} \mid \rho_{ij} - 1e^{-5})\right]\right\}}{N \cdot 2e^{-5}} \tag{32}$$

Eqs.(25)-(32) are substituted into Eq.(24), and the solution for $\partial P_F(\boldsymbol{\rho})/\partial\rho_{ij}$ is obtained. Then the sensitivity index ϵ_ρ of the correlation coefficient to the failure probability can be obtained. Next, an example is used to demonstrate the accuracy of the proposed method by comparing it with Monte Carlo method.

5. EXAMPLES

The example considers a two-dimensional functional function as follows:

$$G(X) = b - x_1 x_2 \tag{33}$$

where $b = 15$ is the threshold; and x_1 and x_2 are the random variable. In the performance function, the weights of x_1 and x_2 are the same. The mean value (μ) of x_1 is 7.2, and the coefficient of variation COV of x_1 is 0.15. The mean value (μ) of x_2 is 2.5, and the coefficient of variation COV of x_2 is 0.13. The correlation coefficient ρ between x_1 and x_2 is varied within $[-0.8, -0.6, -0.4, -0.2, 0.2, 0.4, 0.6, 0.8]$; And then, the proposed method is calculated 1000 times through Latin hypercube sampling(LHS), and 10^7 times through MCS.

To evaluate the performance of the proposed method for general cases, x_1 and x_2 are all assumed to follow Normal distributions, and the copula functions assumed to be the Gaussian copula and Clayton copula, the expressions for the copula functions are shown in Eq.(35) and Eq.(36). In these two cases, the failure probability and correlation coefficient sensitivity indexes are shown in Figure 1 and Figure 2, respectively. The proposed method is in good agreement with MCS under low failure probability conditions, with small errors. When the correlation coefficient is 0, the sensitivity index is also 0, which indirectly proves the accuracy of the results of the proposed method.

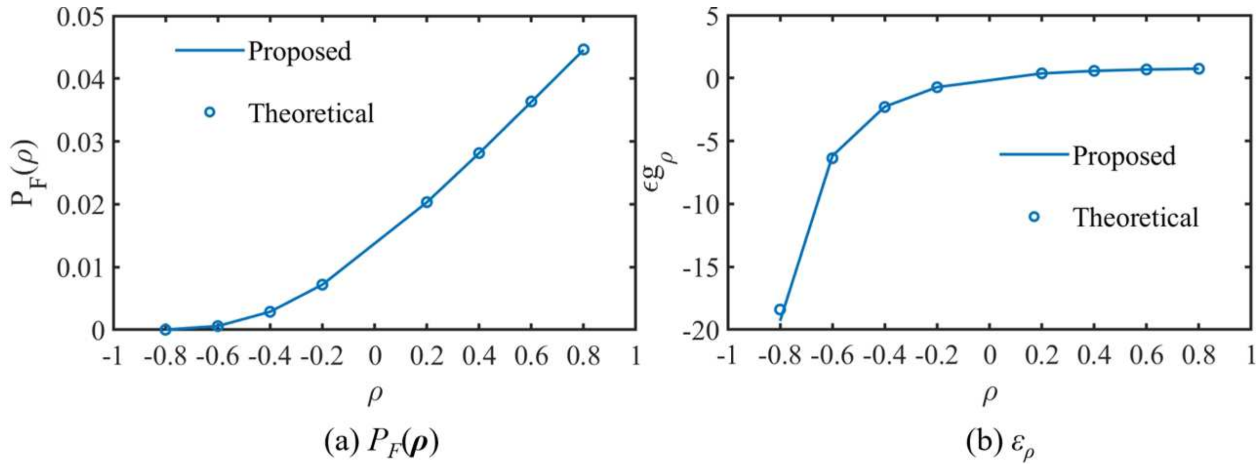

Figure 1. Gauss distribution and Gauss copula

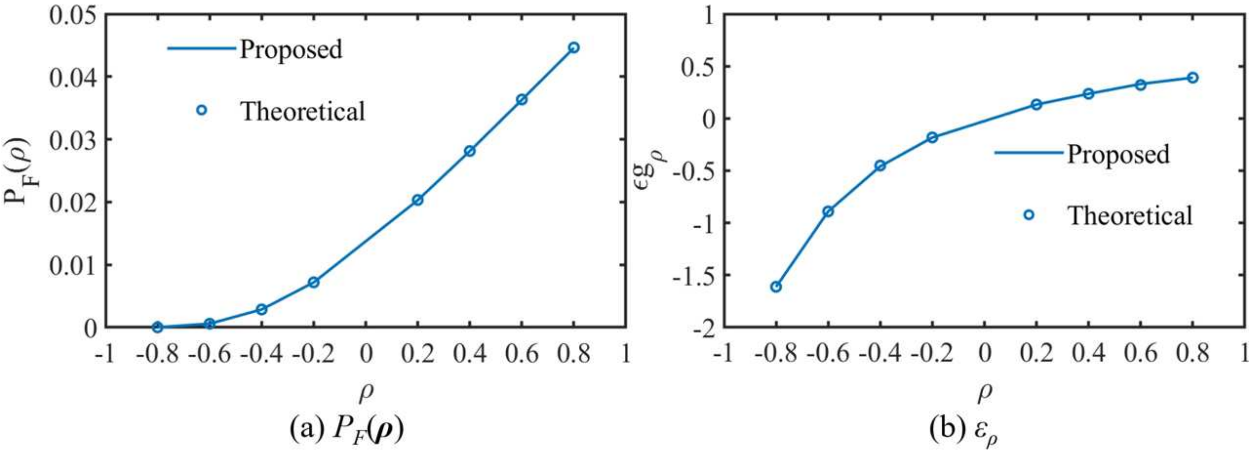

Figure 2. Gauss distribution and Clayton copula

6. Discussion and Conclusions

A reliability sensitivity analysis method based on the first four L-moments is proposed in this paper, considering parameter correlation. As a byproduct of reliability analysis, the proposed method requires no additional calculations, greatly facilitating sensitivity analysis. The example shows that the sensitivity of the correlation coefficient of the input random variable to the failure probability obtained by the proposed method is highly consistent with the MCS results, which fully demonstrates the effectiveness of the proposed method.

It should also be pointed out that the proposed method uses the first four moments of the functional function, so it is only applicable when these moments are sufficient to describe the statistical properties of the functional function.

Appendix

Complete monotonic expressions of normal transformation based on the first four L-moments is shown in Table.(I).

$$Y = a + bU + cU^2 + dU^3 \tag{34}$$

Table I. Y-U monotonic transformation expression based on linear moments

Linear moment ratio			U-expression	**X-Range**
$\tau_4 \neq \delta_4$	$\tau_3^2 \leq \delta_3$		$\sqrt[3]{-\hbar/2-\sqrt{A}}+\sqrt[3]{\hbar/2+\sqrt{A}}-c/3d$	$(-\infty,+\infty)$
$\tau_4 > \delta_4$	$\tau_3^2 > \delta_3$	$\tau_3 \geq 0$	$2\sqrt{\Delta_0/9d^2}\cos(\alpha/3)-c/3d$	$Q_1^* < x < Q_2^*$
			$\sqrt[3]{-\hbar/2-\sqrt{A}}+\sqrt[3]{-\hbar/2+\sqrt{A}}-c/3d$	$x \geq Q_2^*$
		$\tau_3 < 0$	$\sqrt[3]{-\hbar/2-\sqrt{A}}+\sqrt[3]{-\hbar/2+\sqrt{A}}-c/3d$	$x \leq Q_1^*$
			$-2\sqrt{\Delta_0/9d^2}\cos(\alpha/3)-c/3d$	$Q_1^* < x < Q_2^*$
$\tau_4 < \delta_4$	$\tau_3^2 > \delta_3$		$-2\sqrt{\Delta_0/9d^2}\cos(\alpha/3)-c/3d$	$Q_2^* \leq x \leq Q_1^*$
$\tau_4 = \delta_4$		$\tau_3 > 0$	$\frac{\sqrt{b^2-4c(a-x)}-b}{2c}$	$x \geq Q_0^*$
		$\tau_3 > 0$	$\frac{\sqrt{b^2-4c(a-x)}-b}{2c}$	$x \leq Q_0^*$
		$\tau_3 = 0$	$\frac{x-a}{b}$	$(-\infty,+\infty)$

The calculation formulas in the chart can be found in the literature(Zhao et al., 2020; Tong et al., 2021; Tong et al., 2025). The commonly used copula functions gaussian copula and clayton copula are as follows:

$$c(u,v) = \frac{1}{\sqrt{|\rho|}} \exp\left(-0.5 \cdot z^T(\rho^{-1} - I)z\right) \tag{35}$$

$$c(u,v;\theta) = (1+\theta)(uv)^{-1-\theta}\left(u^{-\theta} + v^{-\theta} - 1\right)^{-2-1/\theta} \tag{36}$$

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