An Efficient Reliability Sensitivity Analysis Method Based on the Method of Moments

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Abstract. With the growing demand for refined structural reliability assessments, the influence of higher-order moments on failure probability has garnered increasing attention. However, existing third-order sensitivity methods (SAMoM) cannot reveal the sensitivity of failure probability to fourth-order moments. This paper proposes a fourth-order moment sensitivity analysis method that computes the partial derivatives of failure probability with respect to the mean, standard deviation, skewness, and kurtosis of input variables. Numerical example show that the method maintains the accuracy of SAMoM while, for the first time, reliably estimating the sensitivity of failure probability to kurtosis.

Keywords: Sensitivity analysis; Failure probability; Method of methods; First-four moments.

1. Introduction

Reliability sensitivity analysis holds a crucial role in structural design optimisation, risk assessment and uncertainty management. By quantifying the response of failure probabilities to changes in the distribution parameters of input random variables, sensitivity analysis provides the necessary gradient information for reliability-based design optimisation and helps to identify the most influential parameters(Ditlevsen and Madsen, 1996; Lemaire, 2013; Der Kiureghian, 2022).Both global and local sensitivity measures have been developed in the field of engineering.

Global sensitivity analyses, such as the Sobol' indices(Sobol', 1990; 2001), are used to assess the average impact of input variables across their entire range of variation; however, they do not indicate whether the impact is positive or negative, and involve significant computational effort when the probability of failure is low. Local sensitivity analysis is defined as the partial derivative of the probability of failure with respect to a distribution parameter; this type of analysis is better suited to gradient-based optimisation and can be obtained efficiently as a by-product of many reliability methods(Wei et al., 2022).

Among existing local sensitivity analysis methods, the Method of Moments (MoM)(Zhao and Ono, 2001) strikes a balance between accuracy and efficiency. MoM approximates the probability of failure using the first few statistical moments of the performance function, thereby avoiding repeated evaluations of the original

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model.The third-order moment sensitivity analysis method (SAMoM) has been proposed(Zhang et al., 2025), which is capable of estimating the sensitivity of the probability of failure to the mean, standard deviation and skewness.

However, Many engineering problems involve input variables with heavy-tailed distributions; in such cases, kurtosis significantly influences tail behaviour(Li et al., 2023), thereby affecting the probability of rare failure events. Both SAMoM and other existing methods fail to quantify the independent contribution of kurtosis to the probability of failure.

To overcome this limitation, researchers have developed a reliability analysis method based on fourth-order moments(Zhao et al., 2018; Zhang et al., 2019).Based on fourth-order central moments, this paper derives a complete chain rule expression for the sensitivity of the failure probability to the first four moments of any independent input variable. A fourth-order moment reliability index is employed, and the central difference method is used to calculate its partial derivatives with respect to each central moment, thereby avoiding overly complex analytical expressions.

2. Proposed method

2.1. Failure probability and method of moments

Consider a structural system whose performance is characterized by a continuous random vector $\mathbf{X} = (X_1, X_2, \cdots, X_n)^T$ of dimension n, representing the uncertain input variables such as material properties, geometric dimensions, and external loads. The system behavior is described by a performance function $G(\mathbf{X})$, where the event of failure is defined as the set of outcomes for which the performance function takes non-positive values:

$$F = \{\mathbf{X} : G(\mathbf{X}) \leq 0\}, \tag{1}$$

Given that $\Omega_{\mathbf{X}} \subseteq \mathbb{R}^n$ denote the full domain of the input variables $\mathbf{X}$, i.e., the n-dimensional space of all possible values (typically the entire $\mathbb{R}^n$, or a subset defined by physical constraints). The probability of failure, denoted by $P_f(\boldsymbol{\theta})$, is given by the integral of the joint probability density function (JPDF), denoted by $f_{\mathbf{X}}(\mathbf{x}|\boldsymbol{\theta})$, over the failure domain as follow:

$$P_f(\boldsymbol{\theta}) = \int_{G(\mathbf{X})\leq 0} f_{\mathbf{X}}(\mathbf{x}|\boldsymbol{\theta}) d\mathbf{x} = \int_{\Omega_{\mathbf{X}}} I_F(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}|\boldsymbol{\theta}) d\mathbf{x}, \tag{2}$$

where $\boldsymbol{\theta}$ is a vactor of the moments of $\mathbf{X}$ and $I_F(\mathbf{x})$ is the indicator function of the failure domain, taking the value 1 if $G(\mathbf{X}) \leq 0$ and 0 otherwise. With $P_f(\boldsymbol{\theta})$ obtained, the reliability index can be obtained as:

$$\beta(\boldsymbol{\theta}) = -\Phi^{-1}[P_f(\boldsymbol{\theta})], \tag{3}$$

where $\Phi[\cdot]$ is the cumulative distribution function (CDF) of standard normal distribution.

In general, it is difficult to compute multiple integrals directly. The method of moments (MoM) provides an efficient approximation scheme, treating the $G(\mathbf{X})$ as a random variable Z and expressing $P_f(\boldsymbol{\theta})$ in terms of:

$$P_f(\boldsymbol{\theta}) = F_G[0|\boldsymbol{\theta}_G(\boldsymbol{\theta})], \tag{4}$$

where $F_G(\cdot|\boldsymbol{\theta}_G)$ is the CDF of $Z = G(\mathbf{X})$, and $\boldsymbol{\theta}_G(\boldsymbol{\theta})$ is the vector of the moments of $G(\mathbf{X})$.

According to MoM, the failure probability can also be expressed as:

$$P_f(\boldsymbol{\theta}) = \Phi[-\beta_{4M}(\boldsymbol{\theta}_G)], \tag{5}$$

where $\beta_{4M}(\boldsymbol{\theta}_G)$ is the fourth order reliability index, the general form of which is shown below:

$$\beta_{4M}(\boldsymbol{\theta}_G) = \sqrt[3]{D_0 + q_0} - \sqrt[3]{D_0 - q_0} + \frac{\tilde{h}_3}{3\tilde{h}_4} \tag{6}$$

where $D_0, q_0, \tilde{h}_3, \tilde{h}_4$ are the parameters parameters that depend on the skewness and kurtosis; detailed expressions are given in the Appendix.

To evaluate β_{4M}, the first four central moments $\theta_G^{(i)}(i = 1, 2, 3, 4)$ of the performance function are required, defined as:

$$\theta_G^{(1)} = \mathbb{E}_{G1}(\boldsymbol{\theta}), \tag{7}$$

$$\theta_G^{(2)} = \sqrt{\mathbb{E}_{G2}(\boldsymbol{\theta}) - [\mathbb{E}_{G1}(\boldsymbol{\theta})]^2}, \tag{8}$$

$$\theta_G^{(3)} = \frac{1}{\sigma_G^3}\{\mathbb{E}_{G3}(\boldsymbol{\theta}) - 3\mathbb{E}_{G1}(\boldsymbol{\theta})\mathbb{E}_{G2}(\boldsymbol{\theta}) + 2[\mathbb{E}_{G1}(\boldsymbol{\theta})]^3\}, \tag{9}$$

$$\theta_G^{(4)} = \frac{1}{\sigma_G^4}\{\mathbb{E}_{G4}(\boldsymbol{\theta}) - 4\mathbb{E}_{G1}(\boldsymbol{\theta})\mathbb{E}_{G3}(\boldsymbol{\theta}) + 6[\mathbb{E}_{G1}(\boldsymbol{\theta})]^2\mathbb{E}_{G2}(\boldsymbol{\theta}) - 3[\mathbb{E}_{G1}(\boldsymbol{\theta})]^4\}, \tag{10}$$

where $\mathbb{E}_{Gi}(\boldsymbol{\theta})$ is the *i*th raw moment of the performance function $G(\mathbf{X})$, which can be theoretically computed from $\boldsymbol{\theta}$ as follows:

$$\mathbb{E}_{Gi}(\boldsymbol{\theta}) = \int_{\Omega_{\mathbf{x}}} [G(\mathbf{x})]^i f_{\mathbf{X}}(\mathbf{x}|\boldsymbol{\theta}) d\mathbf{x}, \quad (i = 1, 2, 3, 4), \tag{11}$$

Substituting Eqs. (7)-(10) into Eqs. (5) and (6) yields the moment-based estimate of the failure probability. Consequently, the method of moments transforms the original multi-dimensional integral over the failure domain into the much simpler task of computing the first four moments of the performance function, followed by closed-form expressions for the failure probability, thus avoiding the discontinuity introduced by the indicator function.

2.2. Sensitivity index of failure probability

Reliability sensitivity analysis aims to quantify how small changes in the distribution parameters θ affect the failure probability. The sensitivity measure adopted in this work is the partial derivative of the failure probability with respect to the distribution parameter:

$$\epsilon_{\theta_{ij}} = \frac{\partial P_f(\boldsymbol{\theta})}{\partial \theta_{ij}}. \tag{12}$$

where θ_{ij} is the *j*th moment of x_i.

In this study, the input random variables $x_i(i = 1, 2, \cdots, n)$ are assumed to be mutually independent, which allows the JPDF to factorize as $f_{\mathbf{X}}(\mathbf{x}) = \prod_{i=1}^{n} f_{X_i}(x_i|\boldsymbol{\theta}_i)$, where $f_{X_i}(x_i|\boldsymbol{\theta}_i)$ is the marginal PDF of x_i, and $\boldsymbol{\theta}_i = (\theta_{i1}, \theta_{i2}, \theta_{i3}, \theta_{i4})^T$ denotes the vector of distribution parameters for x_i. Then, the partial derivative of $P_f(\boldsymbol{\theta})$ with respect to θ_{ij} can be expressed as(Wu, 1994):

$$\frac{\partial P_f(\boldsymbol{\theta})}{\partial \theta_{ij}} = \int_{G(\mathbf{X})\leq 0} h_{ij}(x_i|\boldsymbol{\theta}_i) f_{\mathbf{X}}(\mathbf{x}|\boldsymbol{\theta}) d\mathbf{x}, \tag{13}$$

where the function $h_{ij}(X_i|\boldsymbol{\theta}_i)$ is defined as the score function:

$$h_{ij}(X_i|\boldsymbol{\theta}_i) = \frac{1}{f_{X_i}(X_i|\boldsymbol{\theta}_i)} \cdot \frac{\partial f_{X_i}(X_i|\boldsymbol{\theta}_i)}{\partial \theta_{ij}}. \tag{14}$$

As can be seen from Eqs. (13) and (14), the computation of both the failure probability and its sensitivity $\partial P_f(\boldsymbol{\theta})/\partial \theta_{ij}$ involves a multi-fold integral over the failure domain. Direct evaluation of these integrals is challenging when the performance function $G(\mathbf{X})$ is implicit, highly nonlinear, or when the dimension is large. To overcome this difficulty, method of moment approximate these integrals using the first few statistical moments of $G(\mathbf{X})$, thereby avoiding repeated evaluations of the original performance function. The present work extends this moment-based framework to incorporate fourth-order moment information, enabling the estimation of sensitivity with respect to kurtosis.

Based on Eqs.(5)-(11) and using the chain rule of differentiation, the derivative of $P_f(\boldsymbol{\theta})$ with respect to θ_{ij} is obtained as follows:

$$\frac{\partial P_f(\boldsymbol{\theta})}{\partial \theta_{ij}} = \frac{\partial P_f(\boldsymbol{\theta})}{\partial \beta_{4M}(\boldsymbol{\theta}_G)} \cdot \frac{\partial \beta_{4M}(\boldsymbol{\theta}_G)}{\partial \boldsymbol{\theta}_G} \cdot \frac{\partial \boldsymbol{\theta}_G}{\partial \vec{\mathbb{E}}_G(\boldsymbol{\theta})} \cdot \frac{\partial \vec{\mathbb{E}}_G(\boldsymbol{\theta})}{\partial \theta_{ij}}. \tag{15}$$

There are four elements in Eq. (15), which will be discussed separately in the following. Based on Eq. (5), the derivative of failure probability with respect to the fourth moment-based reliability index is obtained as follows:

$$\frac{\partial P_f(\boldsymbol{\theta})}{\partial \beta_{4M}(\boldsymbol{\theta}_G)} = -\varphi[-\beta_{4M}(\boldsymbol{\theta}_G)], \tag{16}$$

where $\varphi[\cdot]$ is the PDF of the standard normal distribution.

The fourth-moment reliability index β_{4M} is implicitly defined by Eq. (6) together with the parameters Δ, q, h_3, h_4 given in the Appendix. The analytical partial derivatives of β_{4M} with respect to $\theta_G^{(1)}$, $\theta_G^{(2)}$, $\theta_G^{(3)}$, $\theta_G^{(4)}$ are extremely lengthy and cumbersome to derive and implement. To avoid complicated analytical manipulations and to ensure numerical stability, this work computes these gradients using the central finite difference method.

For the *i*-th component $\theta_G^{(i)}$ of $\boldsymbol{\theta}_G$, the first-order partial derivative is approximated as:

$$\frac{\partial \beta_{4M}}{\partial \theta_G^{(i)}} \approx \frac{\beta_{4M}(\theta_G^{(i)} + \delta) - \beta_{4M}(\theta_G^{(i)} - \delta)}{2\delta}, \tag{17}$$

where the step size δ is set to $\delta = 10^{-6} \cdot |\theta_G^{(i)}|$; when $|\theta_G^{(i)}| < 10^{-6}$, an absolute step $\delta = 10^{-6}$ is used.

Each partial derivative requires only two additional evaluations of β_{4M}, which involves only elementary algebraic operations and incurs negligible computational cost. In all numerical examples, this step size yields relative errors below 10^{-10}, which is more than sufficient for engineering accuracy.

The term $\partial\boldsymbol{\theta}_G/\partial\vec{\mathbb{E}}_G(\boldsymbol{\theta})$ is a 4×4 matrix, which is obtained based on Eqs. (7)-(10) as follows:

$$\frac{\partial\boldsymbol{\theta}_G}{\partial\vec{\mathbb{E}}_G(\boldsymbol{\theta})}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ -\dfrac{\theta_G^{(1)}}{\theta_G^{(2)}} & \dfrac{1}{2\theta_G^{(2)}} & 0 & 0 \\ \dfrac{3\mathbb{E}_{G3}\mathbb{E}_{G1}-3\mathbb{E}_{G2}^2}{{\theta_G^{(2)}}^5} & \dfrac{3(\mathbb{E}_{G1}\mathbb{E}_{G2}-\mathbb{E}_{G3})}{2{\theta_G^{(2)}}^5} & \dfrac{1}{{\theta_G^{(2)}}^3} & 0 \\ \dfrac{\partial\theta_G^{(4)}}{\partial\mathbb{E}_{G1}} & \dfrac{\partial\theta_G^{(4)}}{\partial\mathbb{E}_{G2}} & -\dfrac{4\mathbb{E}_{G3}}{{\theta_G^{(2)}}^5} & \dfrac{1}{{\theta_G^{(2)}}^4} \end{pmatrix}. \tag{18}$$

and,

$$\frac{\partial\theta_G^{(4)}}{\partial\mathbb{E}_{G1}} = \frac{4\mathbb{E}_{G1}\mathbb{E}_{G4}+12\mathbb{E}_{G1}\mathbb{E}_{G2}^2-12\mathbb{E}_{G1}^2\mathbb{E}_{G3}-4\mathbb{E}_{G2}\mathbb{E}_{G3}}{2{\theta_G^{(2)}}^5} \tag{19}$$

$$\frac{\partial\theta_G^{(4)}}{\partial\mathbb{E}_{G2}} = \frac{-2\mathbb{E}_{G4}+8\mathbb{E}_{G1}\mathbb{E}_{G3}-6\mathbb{E}_{G1}^2\mathbb{E}_{G2}}{2{\theta_G^{(2)}}^5} \tag{20}$$

Based on Eq. (13) and Eq. (11), the term $\partial\mathbb{E}_{Gk}(\boldsymbol{\theta})/\partial\theta_{ij}$ (k=1, 2, 3, 4) can be expressed as (Zhang et al., 2025):

$$\frac{\partial E_{Gk}(\boldsymbol{\theta})}{\partial\theta_{ij}}=\int_{\Omega_{\mathbf{x}}} H_{ijk}(\mathbf{x})f_{\mathbf{X}}(\mathbf{x}|\boldsymbol{\theta})d\mathbf{x}, \tag{21}$$

where $H_{ijk}(\mathbf{X})$ is a function of $\mathbf{X}$ formulated as follows:

$$H_{ijk}(\mathbf{X})=[G(\mathbf{X})]^k h_{ij}(x_i|\boldsymbol{\theta}_i). \tag{22}$$

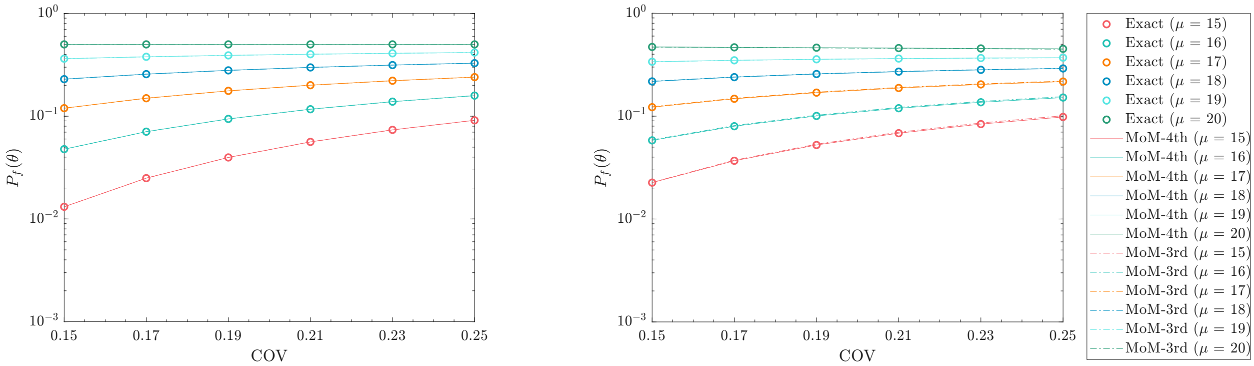
It can be seen from the Eqs. (21) and (22) that the final term is essentially a high-dimensional integral, with the integrand being . To avoid performing high-dimensional numerical integration directly, this paper employs the point estimate method (Zhao and Ono, 2000) combined with the bivariate dimension reduction method (BDRM) (Xu and Rahman, 2004) for approximate calculation.

3. Illustrative example

A one-dimentional performance function are considered as follow:

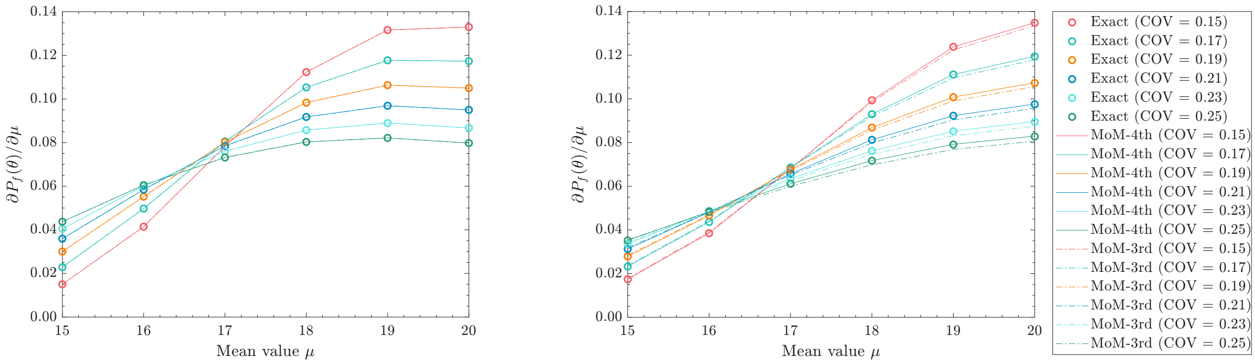
$$G(\mathbf{X})=b-X, \tag{23}$$

where b=20 is the threshold parameter; and X is a random variable. To assess the accuracy of the proposed method, X is assumed to be normal and lognormal distribution.The mean value μ is set to be (15, 16, 17, 18, 19, 20) and the coefficient of variation (COV) is set to be (0.15, 0.17, 0.19, 0.21, 0.23, 0.25).



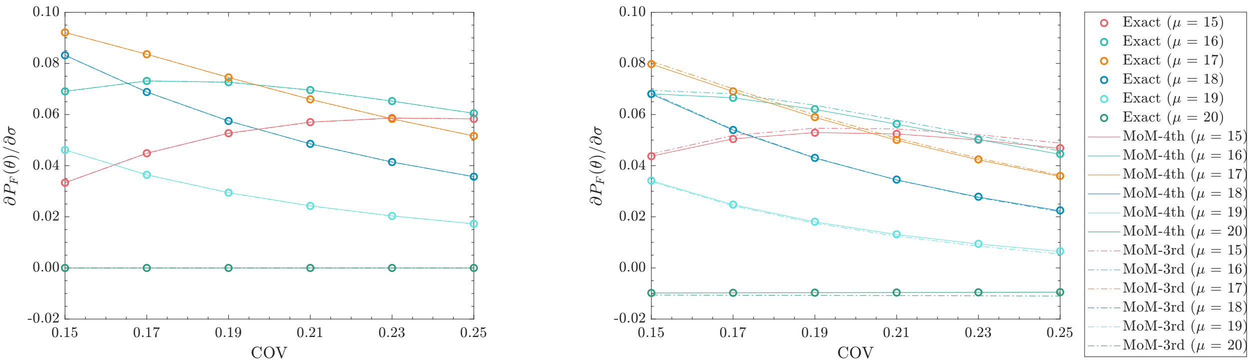
(a)Normal distribution (b)Lognormal distribution

Figure 1. Failure probability for different cases in example



(a)Normal distribution (b)Lognormal distribution

Figure 2. Changing of $\partial P_f(\theta)/\partial\mu$ with different mean values in example



(a)Normal distribution (b)Lognormal distribution

Figure 3. Changing of $\partial P_f(\theta)/\partial\sigma$ with COV in example

Table I. .

random variable	distribution	μ	COV	skewness	kurtosis
X	Pearson system	14	[0.10 0.15 0.20]	-0.1	[2.8 2.9 3.0 3.1 3.2]

Table II. Example of multi-column subcolumns.

COV	skewness	kurtosis	ϵ_3		ϵ_4	
			Proposed method	Exact	Proposed method	Exact
0.10			0.0000	0.0000	0.0000	0.0000
0.15	-0.1	2.8	0.0043	0.0065	0.0019	0.0041
0.20			0.0082	0.0318	0.0072	0.0105
0.10			0.0000	0.0000	0.0000	0.0000
0.15	-0.1	2.9	0.0073	0.0074	0.0041	0.0041
0.20			0.0289	0.0293	0.0085	0.0087
0.10			0.0000	0.0000	0.0000	0.0000
0.15	-0.1	3.0	0.0078	0.0078	0.0039	0.0039
0.20			0.0269	0.0273	0.0072	0.0073
0.10			0.0001	0.0001	0.0001	0.0001
0.15	-0.1	3.1	0.0080	0.0080	0.0036	0.0037
0.20			0.0252	0.0256	0.0061	0.0063
0.10			0.0002	0.0002	0.0001	0.0001
0.15	-0.1	3.2	0.0081	0.0081	0.0034	0.0035
0.20			0.0238	0.0242	0.0053	0.0055

In these two cases, the failure probability and sensitivity indexes are shown in Figure 1 to Figure 3, respectively.

To verify the sensitivity of the proposed method to skewness and kurtosis, the Pearson distribution is assumed to be another distribution type for the random variable X, and the distribution parameters are shown in Table I. In this case, the results of sensitivity index are shown in Table II.

After comparison, the error between the proposed method and the true values was generally within 5%; however, there were still some outliers that differed from the true values, and these should be optimised in future research.

4. Conclusion

This paper proposes a reliability sensitivity analysis method based on the first four central moments. This method treats sensitivity analysis as a by-product of reliability analysis, eliminating the need for additional calculations and thereby greatly simplifying the sensitivity analysis process. Examples demonstrate that the sensitivity of failure probability to the parameters of the random variable distribution obtained using this method is highly consistent with the results derived from exact calculations, thereby fully validating the effectiveness of the method.

This is the first time that the results of a sensitivity analysis of fourth-order moments have been published, thereby quantifying the impact of kurtosis.

Appendix

The parameters for the fourth order moment reliability index are as follows:

$$D_0 = \sqrt{q_0^2 + p_0^3}, \tag{24}$$

and,

$$q_0 = \frac{\tilde{h}_3^3}{27\tilde{h}_4^3} - \frac{\tilde{h}_2\tilde{h}_3}{6\tilde{h}_4^2} - \frac{\tilde{h}_3 - \beta_{2M}/k}{2\tilde{h}_4}, \quad p_0 = \frac{3\tilde{h}_2\tilde{h}_4 - \tilde{h}_3^2}{9\tilde{h}_4^2}, \tag{25}$$

where $\tilde{h}_3$, $\tilde{h}_4$, $\tilde{h}_0$ and k are the parameters calculated based on $\theta_G^{(3)}$ and $\theta_G^{(4)}$; β_{2M} is the so-called second order reliability index and is formulated as follows:

$$\begin{aligned} \tilde{h}_3 &= \frac{5 + (35 - {\theta_G^{(3)}}^2)\tilde{h}_4^2}{9\tilde{h}_0 + 30 - 0.8{\theta_G^{(3)}}^2}\theta_G^{(3)}, \quad \tilde{h}_4 = \frac{2\tilde{h}_0}{2\tilde{h}_0 + 46(1 - 1/{\theta_G^{(4)}}^2) - {\theta_G^{(3)}}^2}, \\ \tilde{h}_0 &= \frac{\sqrt{3\theta_G^{(4)} - 4{\theta_G^{(3)}}^2 - 5} - 2}{1 - (3{\theta_G^{(3)}}^2 + 1)/{\theta_G^{(4)}}^2}, \quad k = \frac{1}{\sqrt{1 + 2\tilde{h}_3^2 + 6\tilde{h}_4^2}}. \end{aligned} \tag{26}$$

$$\beta_{2M} = \frac{\theta_G^{(1)}}{\theta_G^{(2)}}. \tag{27}$$

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