A Bayesian Framework for Modeling and Testing the Temperature-Dependent Residual Compressive Strength of Concrete

Md Maidul Islam
Ghent University, Technologiepark-Zwijnaarde 60, Ghent 9052, Belgium, MdMaidul.Islam@UGent.be
Andrea Franchini
Ghent University, Technologiepark-Zwijnaarde 60, Ghent 9052, Belgium, Andrea.Franchini@UGent.be
Balsa Jovanovic
Ghent University, Technologiepark-Zwijnaarde 60, Ghent 9052, Belgium, Balsa.Jovanovic@UGent.be
Ruben Van Coile
Ghent University, Technologiepark-Zwijnaarde 60, Ghent 9052, Belgium, Ruben.VanCoile@UGent.be

Abstract: This study presents a novel Bayesian model for the temperature-dependent residual compressive strength of a concrete mix. The model consists of a lognormal distribution for the concrete strength retention factor k_{fc}, i.e., the ratio of the compressive strength after exposure to elevated temperature to the initial compressive strength at room temperature. The Bayesian conceptualization of the model enables straightforward updating of the k_{fc} model as new experimental data becomes available. Furthermore, a framework for pre-posterior analysis is presented to guide decision-making before actual data collection. Specifically, the concept of expected information gain (EIG) is adopted to optimize experimental design. This approach allows the identification of optimal measurement temperatures to reduce uncertainty in estimating k_{fc}. The analysis reveals that measuring at 600–900 °C yields a higher information gain than measuring at any other temperature. Comparative analyses illustrate the effectiveness of the EIG concept in guiding experimental planning.

Keywords: Bayesian updating; experimental design; uncertainty quantification.

1. Introduction

Concrete is one of the most widely used construction materials and is commonly considered to behave well under fire due to its low thermal conductivity (Gernay et al., 2021). Nonetheless, concrete structural elements may require post-fire repair due to the permanent degradation of material properties. Existing literature data on residual concrete strength however exhibit large variability (e.g., Al - Jabri et al., 2016; Chen et al. 2014).

The residual concrete strength is commonly evaluated by specifying a concrete strength retention factor k_{fc}. This retention factor is defined as the residual compressive strength $f_{c,T}$ of a specimen having been exposed to a maximum temperature *T*, divided by the initial room-temperature strength $f_{c,20}$. (Shahraki et al., 2023) adopted the methodology from (Qureshi et al., 2020) to propose k_{fc} models for calcareous and siliceous concretes. The dataset used for the Shahraki models includes a wide range of concrete mixes, thereby representing the variability observed across all the different mixes. When a specific concrete building experiences a fire, adopting the Shahraki model is therefore likely to result in an overestimation of the uncertainty on residual k_{fc}. Moreover, this model does not provide a formal updating mechanism for

© 2026 by the author(s). This work is licensed under the Creative Commons Attribution 4.0 International License (CC BY 4.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

incorporating site-specific new data, which would allow the reduction of uncertainty in k_{fc} for specific buildings.

To address these limitations, a novel temperature-dependent model for the residual compressive strength of concrete is proposed in this paper. The model is developed by reevaluating the dataset presented in (Shahraki et al., 2023) and can be updated with new experimental data through Bayesian techniques (Gelman et al., 2021). These techniques also enable the design of optimal experimental protocols that maximize the expected utility of experiments, as outlined by recent research (Franchini and Van Coile, 2026). Sec. 2.1 of this paper develops the temperature-dependent Bayesian k_{fc} model. Next, Sec. 2.2 outlines the approach to update the model based on newly available information, while Sec. 2.3 presents a methodology to identify the measurement temperature that maximizes expected information gain on k_{fc}. Sec. 3 applies this methodology to determine optimal measurement temperatures for k_{fc} uncertainty reduction. Finally, Sec. 4 provides conclusions.

2. Methodology

2.1. MODEL SETUP

A considerable amount of experimental data on k_{fc} is available in the literature. This study utilizes the data sources outlined in (Shahraki et al., 2023). Because the Shahraki et al. dataset was not compiled in a mix-specific manner, the data sources are reanalyzed and the dataset reconstituted with an indication of the concrete mix. Furthermore, Shahraki et al. adopted an inclusion criterion whereby only samples with a retention time of at least 2 hours were incorporated into the dataset. This inclusion criterion has not been adopted here in order to obtain a larger number of data points. The reevaluated dataset contains data points for 248 different mixes of normal-strength concrete, where each mix is characterized by a temperature-dependent mean strength and standard deviation. This dataset defines the current state of knowledge (prior distribution) regarding k_{fc}.

The temperature-dependent k_{fc} for a particular concrete mix is assumed to be lognormally distributed as per Eq. (1) where μ_{kfc} is the mean and σ_{kfc} is the standard deviation. This assumption is motivated by the common usage of lognormal distributions to describe material strength properties (Holický and Sýkora, 2010) and the characteristic of the lognormal distribution in constraining results to positive values. The quantile levels of k_{fc} are considered to be perfectly correlated across exposure temperatures. For example, the 5th percentile (i.e., 0.05 quantile) of concrete retention factor at 100 °C will decay to a 5th percentile of retention factor at 900°C. The distribution parameters μ_{kfc} and σ_{kfc} (see Eq. (2)-(3)) are considered as uncertain and lognormally distributed with means and standard deviations denoted as μ_μ, σ_μ, μ_σ, and σ_σ.

$$k_{fc} \sim LN(\mu_{kfc}, \sigma_{kfc}^2) \tag{1}$$

$$\mu_{kfc} \sim LN(\mu_\mu, \sigma_\mu^2) \tag{2}$$

$$\sigma_{kfc} \sim LN(\mu_\sigma, \sigma_\sigma^2) \tag{3}$$

The value of k_{fc} at 20 °C is fixed to 1, consistent with its definition as a normalized ratio ($k_{fc} = f_{c,T}/f_{c,20}$). Furthermore, a simplifying assumption is adopted that no residual strength remains after exposure to 1000 °C, resulting in a k_{fc} equal to 0. At both these temperatures, k_{fc} is thus known and has no associated variability.

The variability among multiple mixes is quantified by first considering the mean $\mu_{z,T}$ and standard deviation $\sigma_{z,T}$ data for the z-th mix in the dataset. These statistics are evaluated at selected temperature levels $T \in T_{range} = \{100, 200, 300, 400, 500, 600, 700, 800, 900\}$ [°C], and the results are shown in Figure 1(a). Then, the mean and standard deviation of $\mu_{z,T}$ and $\sigma_{z,T}$ are calculated at each T. This results in 1 data point at each T across T_{range} for the model parameters μ_μ, σ_μ, μ_σ, and σ_σ (as shown in Figure 1(b)-(c)). Finally, polynomial regression is employed to obtain the best-fit curves for the 8 data points across T_{range} (see Eq. (4)-(7) and Figure 1(b)-(c)). The regression is carried out considering the following boundary conditions: $\mu_\mu(20°C) = 1$; $\mu_\mu(1000°C) = 0$; $\sigma_\mu(20°C) = 0$; $\sigma_\mu(1000°C) = 0$; $\mu_\sigma(20°C) = 0$; $\mu_\sigma(1000°C) = 0$; $\sigma_\sigma(20°C) = 0$; $\sigma_\sigma(1000°C) = 0$.

$$\mu_\mu(T) = 1.30 \times 10^{-12} \cdot T^4 - 2.61 \times 10^{-9} \cdot T^3 + 1.43 \times 10^{-6} \cdot T^2 - 1.14 \times 10^{-3} \cdot T + 1.022 \quad (4)$$

$$\sigma_\mu(T) = 2.52 \times 10^{-12} \cdot T^4 - 4.38 \times 10^{-9} \cdot T^3 + 1.17 \times 10^{-6} \cdot T^2 + 6.97 \times 10^{-4} \cdot T - 0.014 \quad (5)$$

$$\mu_\sigma(T) = -5.71 \times 10^{-13} \cdot T^4 + 1.84 \times 10^{-9} \cdot T^3 - 1.99 \times 10^{-6} \cdot T^2 + 7.29 \times 10^{-4} \cdot T - 0.014 \quad (6)$$

$$\sigma_\sigma(T) = -5.20 \times 10^{-13} \cdot T^4 + 1.64 \times 10^{-9} \cdot T^3 - 1.74 \times 10^{-6} \cdot T^2 + 6.27 \times 10^{-4} \cdot T - 0.012 \quad (7)$$

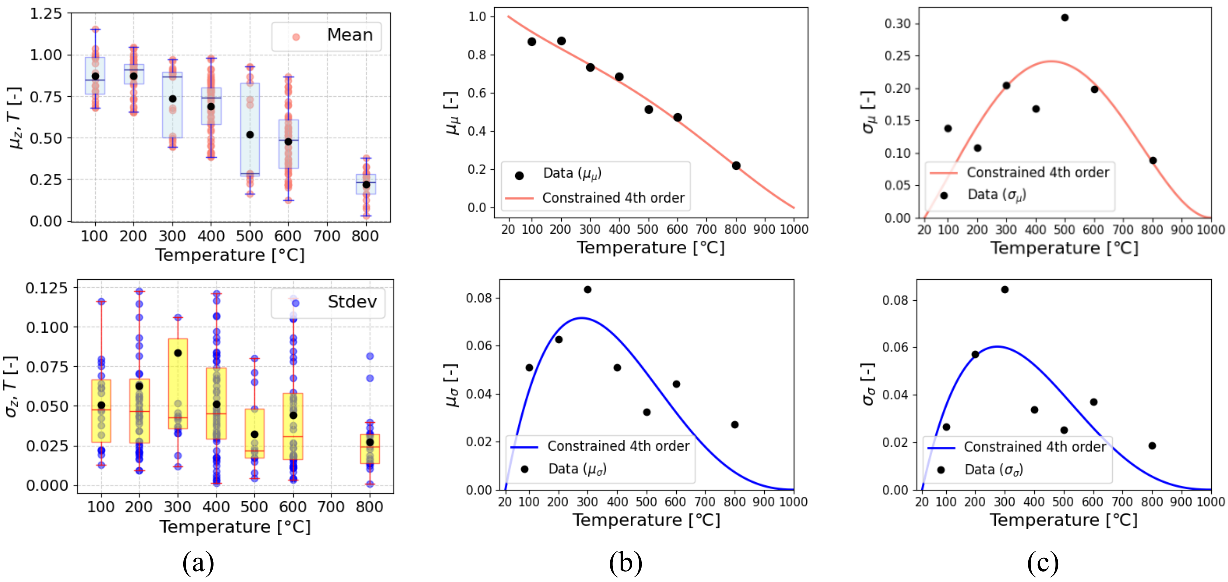


Figure 1. Regression analysis showing: (a) Data points representing 248 different mixes; (b) best-fit polynomials for μ_μ and μ_σ; and (c) best-fit polynomials for σ_μ and σ_σ.

The best-fit values from Eq. (4)-(7) define temperature-dependent distribution parameters for $\mu_{kfc}(T)$ and $\sigma_{kfc}(T)$. The quantile levels within these fixed lognormal distributions are denoted as q_μ and q_σ. For a given realization, the same quantile is assumed to occur at each temperature. Before considering site-specific data, all quantile levels are considered equally likely. Thus, q_μ and q_σ are described by uniform distributions between 0 and 1.

Realizations of k_{fc} are obtained as follows: **(i)** a set of quantile levels q_μ and q_σ is sampled from their uniform distributions using Monte Carlo Simulations (MCS); **(ii)** for each quantile-level pair, a realization pair of $\mu_{kfc}(T)$ and $\sigma_{kfc}(T)$ is obtained from the inverse cumulative lognormal distribution function; **(iii)** random realizations of k_{fc} are drawn at each temperature from the distributions defined by $\mu_{kfc}(T)$ and $\sigma_{kfc}(T)$; **(iv)** step (ii) and (iii) are repeated for each sampled quantile level. The obtained temperature-dependent distribution of k_{fc} is referred to as the "prior predictive distribution".

Figure 2 compares the obtained prior predictive distribution with the Shahraki model for siliceous concrete. Compared to the Shahraki model, the k_{fc} quantile values are closer to 1 after exposure to very low temperatures and closer to 0 at very high temperatures. This aligns with the boundary conditions specified for the model (constraints at 20°C and 1000°C). The proposed model exhibits more uncertainty between 200 and 800 °C than the Shahraki model because the Shahraki model does not consider the uncertainty on μ_{kfc} or σ_{kfc}. Also, a filtered dataset (data points with retaining time > 2 hr only) was used in their work, while all available data is used in this study, resulting in additional uncertainty in the proposed model.

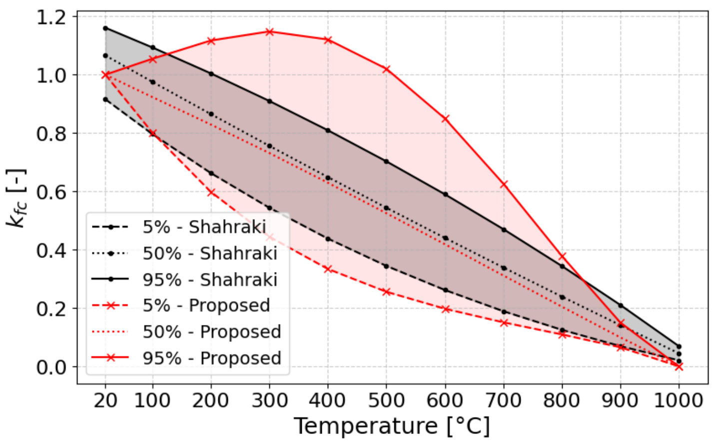


Figure 2. 5%, 50%, and 95% quantiles across T_{range} for the proposed model in comparison with the Shahraki model.

2.2. BAYESIAN MODEL UPDATING

The available knowledge on the parameters of interest $\boldsymbol{\theta}$ (here: q_μ and q_σ defined in Sec. 2.1) is represented through a prior joint probability distribution $p(\boldsymbol{\theta})$. In this paper, the two parameters of interest are assumed to be independent, with priors defined in Sec. 2.1. If an experiment is conducted and data $\boldsymbol{y}$ is observed (here: observations of k_{fc}), Bayes' theorem allows the updating of the prior distribution based on the obtained experimental outcome (Gelman et al., 2021):

$$p(\boldsymbol{\theta}|\boldsymbol{y}) = \frac{p(\boldsymbol{\theta}).p(\boldsymbol{y}|\boldsymbol{\theta})}{p(\boldsymbol{y})} \tag{8}$$

In this equation, $p(\boldsymbol{\theta}|\boldsymbol{y})$ is the posterior probability distribution; $p(\boldsymbol{y}|\boldsymbol{\theta})$ is the likelihood, which gives the probability of observing the data given $\boldsymbol{\theta}$; $p(\boldsymbol{y}) = \sum_{\theta} p(\boldsymbol{\theta})\, p(\boldsymbol{y}|\boldsymbol{\theta})$ is the total probability of observing the data y (also referred to as the marginal likelihood).

When measurements of residual compressive strength are obtained for a specific situation (e.g., post-fire analysis of a specific building), Bayesian analysis combines this data with the information in the prior model and enables updating prior knowledge on k_{fc}. The Markov Chain Monte Carlo (MCMC) method with the Metropolis algorithm (Gelman et al., 2021) can be adopted to solve Eq. (8) numerically and generate posterior samples for q_{μ} and q_{σ}. These samples can then be used to obtain the posterior predictive distribution of k_{fc}, following steps (i)-(iv) from Sec. 2.1.

2.3. OPTIMAL EXPERIMENTAL DESIGN APPROACH

When the goal is to inform decision-making on how to collect residual strength data (e.g., testing temperature, number of measurements), a pre-posterior Bayesian analysis can be carried out. To that end, this study implements the expected utility quantification framework by (Franchini and Van Coile, 2026) and focuses on the expected reduction of uncertainty in estimating the temperature-dependent k_{fc}.

To estimate this reduction, potential experimental observations are first sampled from the prior predictive distribution. Then, for each batch of measurements, Bayesian updating is performed to generate posterior distributions of q_{μ} and q_{σ} as outlined in Sec. 2.2. Finally, posterior predictive distributions of k_{fc} are obtained by following the steps (i)-(iv) described in Sec. 2.1, and information gain (IG) is calculated as the difference between prior and posterior predictive "entropy". Entropy is a measure of uncertainty in a random variable. EIG is the average information gain across all the simulated test batches. Thus, in pre-posterior analysis, if n denotes the number of simulated test batches, the EIG is given by:

$$EIG = \frac{1}{n}\sum_{i=1}^{n} IG_i \tag{9}$$

3. Example application

As an illustrative example, this section aims to identify the measurement temperature at which k_{fc} should be measured when the experimental campaign is limited to 3 measurements. This number of measurements aligns with the predominant sampling size reported in the literature. In this study, the 'evaluation temperature' T_e is defined as the temperature at which the k_{fc} is to be evaluated, which may differ from the temperature at which the experiments are conducted (i.e., the 'measurement temperature', T_m). First, 300 realizations of three potential experimental observations are sampled from the prior predictive distributions (see Figure 3), resulting in an equivalent number of q_μ and q_σ realizations. Because each batch is independently drawn, adopting different pairs of q_μ and q_σ realizations, its corresponding batch-specific strength distribution differs across realizations. This variability represents the uncertainty between different test batches.

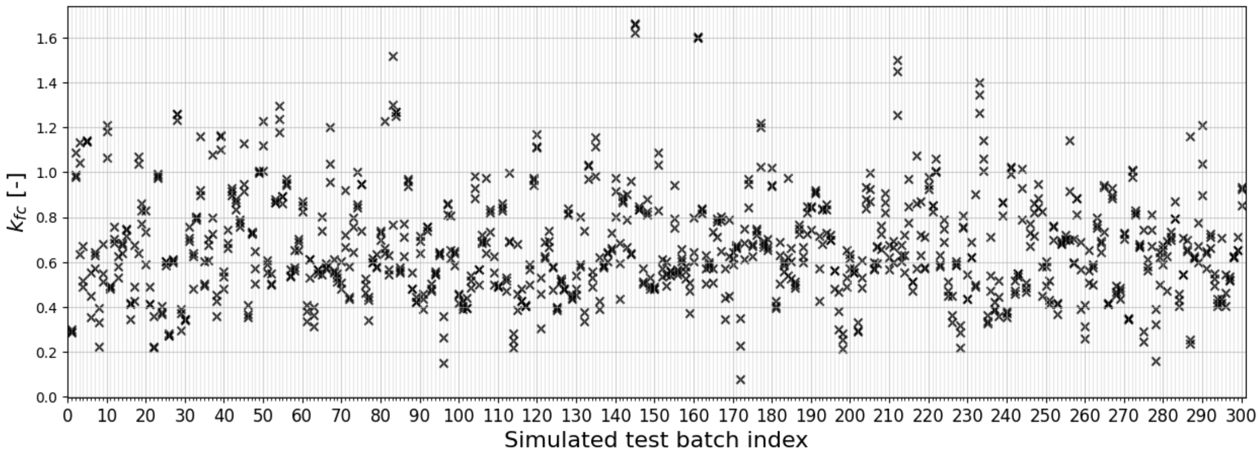
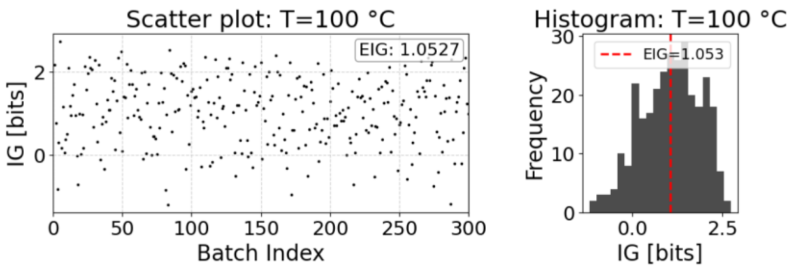
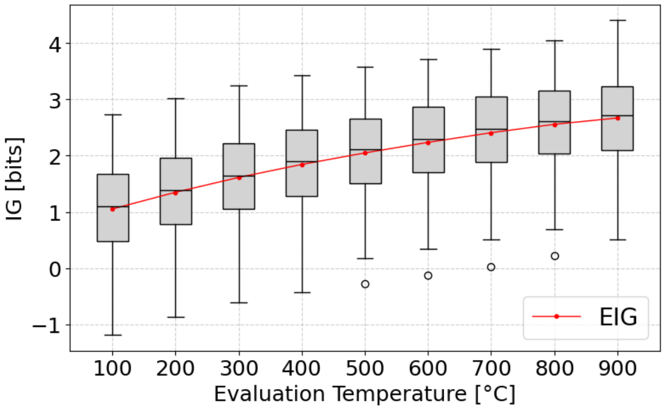


Figure 3. Example of 300 batches of simulated measurements (3 measurements at 400 ℃).

For each simulated test batch, posterior distributions of q_μ and q_σ are generated by employing MCMC simulation. Depending on the convergence of the calculation, 4.5×10^5 to 6×10^5 MCMC simulation draws are generated and used to obtain posterior predictive distributions of k_{fc}. Then, IG values are calculated for each simulated test batch, and their average provides the EIG. The EIG is evaluated at each evaluation temperature $T_e \in T_{range}$. As an example, Figure 4(a) shows the pre-posterior IG and EIG values at T_e = 100 ℃ for 300 simulated test batches of 3 measurements at 400 ℃ (i.e., T_m = 400 ℃). The black dots show IG values for each of the 300 batches, whereas the histogram shows the IG distribution with the mean value (red dashed line) representing the EIG. Figure 4(b) shows the boxplot indicating the mean and interquartile range of the IG across $T_e \in T_{range}$. The red dot at 100 ℃ corresponds to the red dashed line in Figure 4(a) and is the corresponding EIG value. EIG values at other T_e are obtained following the same procedure. The figure also indicates that EIG increases with an increase in evaluation temperature.



(a)



(b)

Figure 4. Plots for 3 measurements at 400 °C showing: (a) Pre-posterior information gain (IG) distributions and corresponding expected information gain (EIG); (b) Information gains (IGs) at evaluation temperatures and their interquartile range.

Figure 5 presents the EIG from three measurements as a function of the selected measurement temperature. Using this plot, a user can select an evaluation temperature and identify the measuring temperature where EIG is the highest. For example, if the estimation of k_{fc} is required at 500 °C, from Figure 5 it can be concluded that measuring at 900 °C results in the highest EIG (approx. 2.37 bits). EIG values obtained for measurements at 600–900 °C are very similar for evaluation temperatures up to 400 °C, indicating that testing within this temperature range provides comparable information gain. However, for evaluations in the 500–900 °C range, the marginal EIG resulting from testing at higher temperatures is notable. Measuring at 100-300 °C results in low EIG for any evaluation temperature. Assessing engineers can use this plot to optimize experimental data collection for the considered site-specific concrete structure.

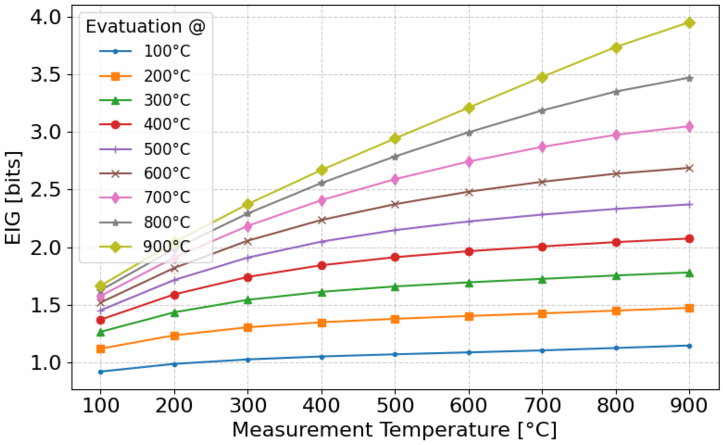


Figure 5. Expected information gain (EIG) across the selected evaluation and measurement temperatures for 3 measurements.

4. Conclusion

The main contributions and findings of this paper can be summarized as follows:

1. A Bayesian model is developed to characterize the temperature-dependent residual compressive strength retention factor for concrete k_{fc}. Unlike existing models, the proposed one captures the variability within a single mix.
2. Capturing the variability within a single mix is beneficial for post-fire assessment, where the goal is to assess a specific structure. The Bayesian formulation of the model allows the incorporation of site-specific experimental data when this becomes available.
3. The model is calibrated using a comprehensive dataset of 248 concrete mixes. To ensure physical consistency, the model enforces a residual retention factor of 1.0 at 20 °C and 0 at 1000°C. These fixed boundary conditions simplify the model specification and allow the variability at 20 °C to be treated independently from the temperature-dependent retention behavior.
4. A pre-posterior analysis is used to identify the most informative experimental protocol by using expected information gain (EIG) as the utility quantification metric. The analysis reveals that measurements within the 600–900 °C range yield significantly higher marginal information gain, highlighting more effective measurement temperatures for model updating and uncertainty reduction.

Acknowledgements

Funded by the European Union (ERC, AFireTest, 101075556). Views and opinions expressed are, however, those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.

The authors gratefully acknowledge the support of prof. Negar Elhami-Khorasani in providing access to the dataset adopted in (Shahraki et al. 2023).

References

Al-Jabri, Khalifa S., Muhammad Bilal Waris, and Abdullah H. Al-Saidy. 2016. "Effect of Aggregate and Water to Cement Ratio on Concrete Properties at Elevated Temperature." *Fire and Materials* 40 (7): 913–25. https://doi.org/10.1002/fam.2351.

Chen, G. M., Y. H. He, H. Yang, J. F. Chen, and Y. C. Guo. 2014. "Compressive Behavior of Steel Fiber Reinforced Recycled Aggregate Concrete after Exposure to Elevated Temperatures." *Construction and Building Materials* 71 (November): 1–15. https://doi.org/10.1016/j.conbuildmat.2014.08.012.

Franchini, Andrea, and Ruben Van Coile. 2026. "Quantifying the Expected Utility of Fire Tests and Experiments before Execution." *Fire Safety Journal* 159 (January): 104538. https://doi.org/10.1016/j.firesaf.2025.104538.

Gelman, Andrew, John B. Carlin, Hal S. Stern, David B. Dunson, Aki Vehtari, and Donald B. Rubin. 2021. *Bayesian Data Analysis (BDA)*. 3rd ed. Electronic edition. https://sites.stat.columbia.edu/gelman/book/BDA3.pdf.

Gernay, Thomas, Venkatesh Kodur, Mohannad Z. Naser, Reza Imani, and Luke Bisby. 2021. "Concrete Structures." In *International Handbook of Structural Fire Engineering*, edited by Kevin LaMalva and Danny Hopkin. Springer International Publishing. https://doi.org/10.1007/978-3-030-77123-2_6.

Holický, Milan, and Miroslav Sýkora. 2010. “Stochastic Models in Analysis of Structural Reliability.” *Proceedings of the International Symposium on Stochastic Models in Reliability Engineering, Life Sciences and Operation Management* (Beer Sheva, Israel).

Qureshi, Ramla, Shuna Ni, Negar Elhami Khorasani, Ruben Van Coile, Danny Hopkin, and Thomas Gernay. 2020. “Probabilistic Models for Temperature-Dependent Strength of Steel and Concrete.” *Journal of Structural Engineering* 146 (6): 04020102. https://doi.org/10.1061/(ASCE)ST.1943-541X.0002621.

Shahraki, Marzieh, Nan Hua, Negar Elhami-Khorasani, Anthony Tessari, and Maria Garlock. 2023. “Residual Compressive Strength of Concrete after Exposure to High Temperatures: A Review and Probabilistic Models.” *Fire Safety Journal* 135 (February): 103698. https://doi.org/10.1016/j.firesaf.2022.103698.