Neural network-driven uncertainty qualification analysis of printed circuit heat exchangers

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Abstract: The randomness in structural parameters of printed circuit heat exchangers (PCHEs) is unavoidable and can significantly impact shakedown limit and reliability. However, studies on the influence of random structural parameters are limited due to the expensive computation burden. Therefore, this paper aims to achieve efficient stochastic shakedown and reliability analyses of PCHEs with multiple random structural parameters. First, the Linear Matching Method (LMM) is utilized to perform the shakedown analysis. Then, to account for the influence of random structural parameters, the Direct Probability Integral Method (DPIM), as an efficient non-intrusive stochastic analysis method, is introduced. By exploiting the advantages of LMM and DPIM, a novel LMM-DPIM is proposed for the uncertainty qualification analysis of PCHEs. Furthermore, the data-driven neural network model is implemented to improve the computational efficiency. As a result, an LMM-DPIM-based neural network (LMM-DPIM-NN) model is established. Finally, the high accuracy and efficiency of the proposed model are verified through comparisons with Monte Carlo simulation. Moreover, the results of uncertainty qualification analysis reveal the effects of different random structural parameters on the shakedown limit and reliability of PCHE. Particularly, the fillet radius at the corner of channels significantly influences the shakedown limit, leading to a huge reduction in reliability.

Keywords: printed circuit heat exchangers; stochastic shakedown and reliability analyses; linear matching method; direct probability integral method; neural network model.

1. Introduction

Printed circuit heat exchangers are compact micro-channel devices widely used in chemical, aerospace, and energy applications owing to their high thermal efficiency and structural compactness [1, 2]. In service, PCHEs are subjected to cyclic thermo-mechanical loads arising from alternating hot and cold fluid channels joined by diffusion bonding. Shakedown analysis is therefore essential for structural integrity assessment [3]: within the shakedown boundary the structure develops limited plastic strain after initial cycles, whereas loads beyond the boundary may lead to low-cycle fatigue or incremental collapse. Although the ASME and R5 codes recommend Bree-diagram-based rule-based methods [4-6], these are restricted to standard geometries;

cycle-by-cycle inelastic finite element analysis, while general, incurs prohibitive computational cost. Several direct methods have been developed for efficient shakedown and limit analysis of engineering structures [7-9]. The Linear Matching Method, grounded in Koiter's kinematic shakedown theorem, has emerged as an accurate and efficient direct method for non-standard components [10-13], and has been successfully applied to deterministic shakedown assessment of PCHEs [14, 15]. However, geometric and material uncertainties inherent in manufacturing—such as channel radius, fillet radius, and temperature dependent yield stress can significantly affect shakedown limits, motivating stochastic analysis as the foundation for reliability-based design.

To quantify these uncertainties, the Direct Probability Integral Method offers a non-invasive framework based on a probability density integral equation that separates the probability evaluation from the physical model [16-18]. Compared with Monte Carlo simulation, DPIM requires far fewer model evaluations and avoids random convergence issues [19]. Nevertheless, when the structural response is strongly nonlinear—as in PCHE shakedown under multiple random parameters—the repeated LMM evaluations needed by DPIM remain costly. Data-driven surrogate models, particularly back-propagation neural networks, can approximate this nonlinear mapping efficiently [20-22] and have shown success in creep-fatigue and ratcheting reliability assessments of pressurized components.

This paper presents an LMM-DPIM-NN framework for stochastic shakedown and reliability analysis of PCHEs. The contributions are: (1) combining LMM with DPIM for the first time and building a neural-network surrogate for PCHEs with multiple random parameters; (2) efficiently obtaining probabilistic shakedown limit information under limited data; and (3) revealing that the fillet radius at the channel corner exerts the strongest influence on the shakedown limit and can trigger stochastic P-bifurcation, with reliability decreasing by up to 19.43 % as the fillet radius increases.

2. Methodology

2.1 Modified LMM for stochastic shakedown analysis

The LMM iteratively updates the shear modulus to match the yield condition based on the plastic strain-rate history, computing the shakedown multiplier λ via Koiter's kinematic theorem. In deterministic analysis, the thermo-mechanical load ratio is fixed and the shakedown multiplier is scaled along a radial line in load space; repeating for different ratios constructs the classical Bree diagram. For stochastic analysis, the load ratio and structural parameters become random variables, and the shakedown boundary transforms from a curve to a surface in the thermo-mechanical load space. The total elastic stress field is decomposed as

$$\sigma_{ij}^{E}(x,t) = \alpha_T \sigma_{ij}^{T}(x,t) + \alpha_P \sigma_{ij}^{P}(x,t) \tag{1}$$

where α_T and α_P are initial multipliers for the construction of various loading combinations, and σ_{ij}^{T} and σ_{ij}^{P} mean the corresponding linear elastic solution. The cyclic stress history is expressed as

$$\Delta\sigma_{ij}(x,t) = \lambda \Delta\sigma_{ij}^{E}(x,t) + \bar{\rho}_{ij}(x,t) + \bar{\rho}_{ij}(x,t) \tag{2}$$

in which $\overline{\rho}_{ij}$ indicates constant residual stress field, λ means the shakedown multiplier to solve, ρ_{ij} is the varying residual stress field of shakedown problems. By introducing Koiter's shakedown theorem, the shakedown multiplier is evaluated by

$$\lambda = \frac{\int_V \sigma_y(T)\Delta\epsilon_{eq}^{p*} dV}{\int_V \Delta\sigma_{ij}^{E}\Delta\epsilon_{ij}^{p*} dV} \tag{3}$$

where $\sigma_y(T)$ is the temperature-dependent yield stress and $\Delta\epsilon_{ij}^{p*}$ is the kinematically admissible plastic strain-rate cycle. By introducing random variables, the deterministic Bree diagram is extended to a two-dimensional Probability Density Function (PDF) surface representing the stochastic shakedown limit, bounded by a 95 %-probability variation envelope.

2.2 LMM-DPIM-based neural network model

DPIM is based on the probability density integral equation (PDIE), derived from the probability conservation principle, where the PDF of response can be obtained by

$$p_{Y_l}(y_l) = \int_{\Omega_\Theta} p_{Y_l|\Theta(y_l|\theta)}\, p_\Theta(\theta)\, d\theta \tag{4}$$

where Θ denotes the random variable vector, Y_l the response of interest, and $p_\Theta(\theta)$ the joint PDF of the inputs. For numerical evaluation, the probability space is partitioned into representative points $\{\theta_q\}$ with assigned probabilities $\{P_q\}$, and the Dirac delta function is replaced by an adaptive Gaussian kernel:

$$p_{Y_l}(y_l) \approx \sum_{q=1}^{N} P_q \cdot \frac{1}{\eta\sqrt{2\pi}} \exp\left[-\frac{\left(y_l - \lambda_l(\theta_q)\right)^2}{2\eta^2}\right] \tag{5}$$

where N is the total number of representative points and η is the smoothing parameter determined adaptively. Reliability is evaluated by introducing the Heaviside function: $R = \sum_{q=1}^{N} P_q \cdot \mathcal{H}\left[\lambda_S(\theta_q) - B\right]$, where B is the prescribed threshold.

The LMM-DPIM-based neural network model (LMM-DPIM-NN) workflow, as illustrated in Figure 1, proceeds in five steps:

• **Step 1.** Partition the probability space to obtain representative points and assigned probabilities as input random variables.

• **Step 2.** Perform LMM-based shakedown analysis for each representative point under varying load ratios to obtain the shakedown multiplier λ.

• **Step 3.** Train a fully connected BPNN to map input random variables to the shakedown multiplier, using training, validation, and test subsets.

• **Step 4.** Solve the PDIE using the trained NN to obtain the PDF of the shakedown multiplier.

• **Step 5.** Conduct reliability analysis based on the shakedown limit threshold and the reliability equation.

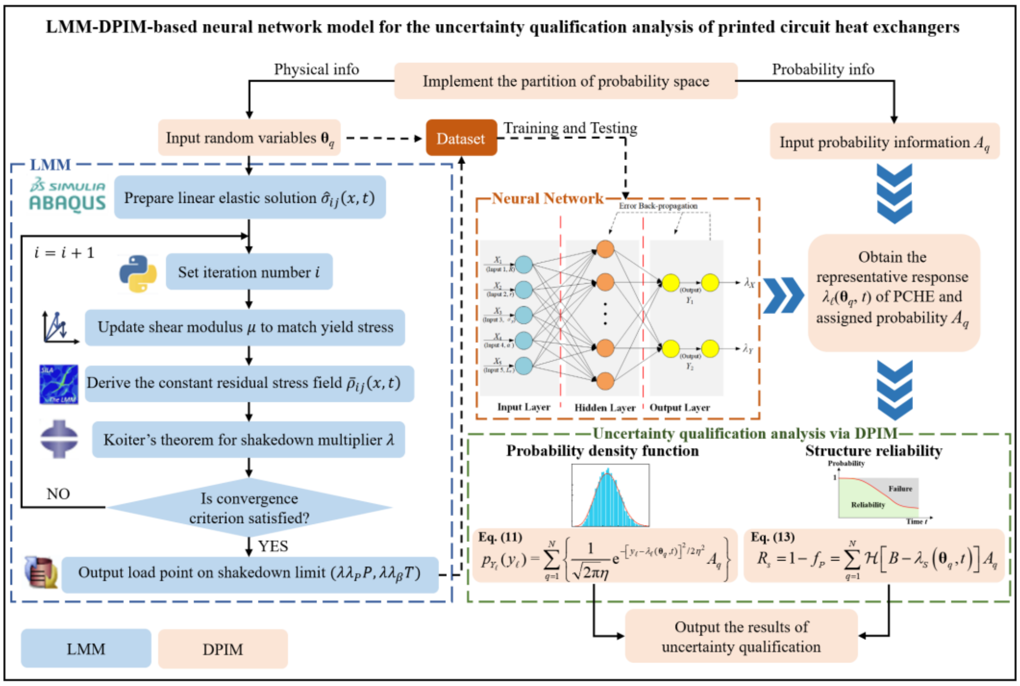


Figure 1. LMM-DPIM-based neural network model for stochastic shakedown and reliability analyses.

3. Numerical example

3.1 PCHE model and random variables

A typical PCHE with semi-circular channels is considered. The hot and cold channel pairs form a unit-cell model. A 2D plane-strain FE model is constructed with symmetric boundary conditions on the symmetry planes and coupled normal displacements on the top and left edges to simulate the repeating channel pattern. The hot channel inner surface is subjected to pressure $P_H = 25$ MPa and temperature $T_H = 50\,°\text{C}$, while the cold channel inner surface carries $P_C = 10$ MPa and $T_C = 20\,°\text{C}$. The FE model is meshed in Abaqus/CAE with 5367 elements.

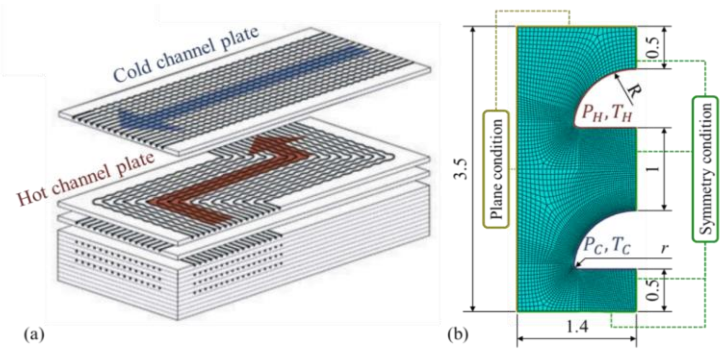


Figure 2. (a) Stacked and bonded plates in a PCHE core; (b) a unitary cell model [14].

The random variables are listed in Table I. Six temperature-dependent yield stresses, channel radius R, fillet radius r, thermal expansion coefficient α, and load ratio L_r are treated as random variables with normal or uniform distributions. Elastic modulus, Poisson's ratio, and thermal conductivity are held constant.

Table I. Random variables with distribution types and ranges.

Design parameter	Temperature [°C]	Mean value (μ_M)	Distribution type	Variation range
Load ratio L_r	—	5	Uniform	[μ_M−5, μ_M+5]
Channel radius R [mm]	—	0.75	Normal	[μ_M−0.1, μ_M+0.25]
Fillet radius r [mm]	—	0.045	Normal	[μ_M−0.08, μ_M+0]
Yield stress σ_y [MPa]	20	241	Normal	[μ_M−12, μ_M+12]
Yield stressσ_y [MPa]	100	212	Normal	[μ_M−12, μ_M+12]
Yield stress σ_y [MPa]	200	188	Normal	[μ_M−12, μ_M+12]
Yield stress σ_y [MPa]	300	174	Normal	[μ_M−12, μ$_M$+12]
Yield stress σ_y [MPa]	400	166	Normal	[μ_M−12,μ_M+12]
Yield stress σ_y [MPa]	500	162	Normal	[μ_M−12, μ_M+12]
Thermal expansion α [10^{-5}/°C]	—	1.83	Normal	[μ_M−0.183, μ_M+0.183]
Young's modulus E [10^3 MPa]	—	159	Constant	—
Poisson's ratio	—	0.3	Constant	—
Thermal conductivity [W/(m·K)]	—	0.02149	Constant	—

3.2 Verification and uncertainty qualification analysis results

Deterministic solution. With all design parameters set to their mean values, the deterministic shakedown limit (Bree diagram) is computed. The upper boundary corresponds to the reverse plasticity limit and the lower boundary to the ratchet limit. Three load points near the shakedown limit are selected for verification by step-by-step analysis in Abaqus with 50 loading–unloading cycles to ensure steady-state cyclic behavior. The PEMAG evolution agrees well with the LMM prediction, confirming the accuracy of the deterministic shakedown limit.

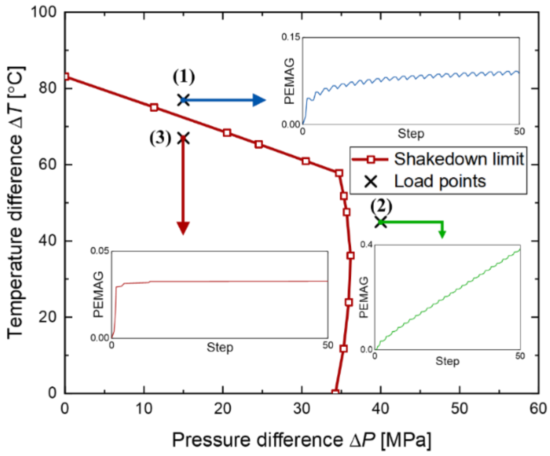


Figure 3. The cyclic behaviour of load points near the deterministic shakedown limit.

LMM-DPIM vs. MCS. Table II compares the LMM-DPIM and MCS results for the stochastic shakedown analysis. The relative error for the CoV of the pressure difference decreases from 268.13% with 100 representative points to 0.43% with 1500 representative points. To guarantee the accuracy of stochastic shakedown analysis, 1500 representative points are selected for LMM-DPIM in this case. Compared with MCS using 1×10^4 samples, LMM-DPIM greatly reduces the CPU time while maintaining good accuracy.

Table II. LMM-DPIM vs. MCS comparison — CoV, relative error, CPU time.

Method	Sampling size	Cov of pressure difference	Relative error	Cov of temperature difference	Relative error	CPU Time
MCS	1×10^4	0.1415	–	0.1127	–	449518.55
LMM-DPIM	100	0.5209	268.13%	0.4514	301.77%	4623.88
LMM-DPIM	500	0.2237	58.09%	0.1823	63.00%	23136.06
LMM-DPIM	1000	0.1460	3.18%	0.1142	1.33%	45921.90
LMM-DPIM	1500	0.1421	0.43%	0.1123	0.35%	69191.72

Stochastic shakedown analysis. Five random parameters (σ_y, R, r, α, L_r) are considered in the Stochastic shakedown analysis via LMM-DPIM with 1500 representative points. Moreover, LMM-DPIM-NN model is established to reproduce the LMM-DPIM results with high accuracy, while reducing CPU time from approximately 68 800 s to 10.7 s. The results of stochastic shakedown are illustrated in Figure 4, 95%-probability shakedown limit variation boundary is indicated by the red line, and the most probable variation range is shown in yellow.

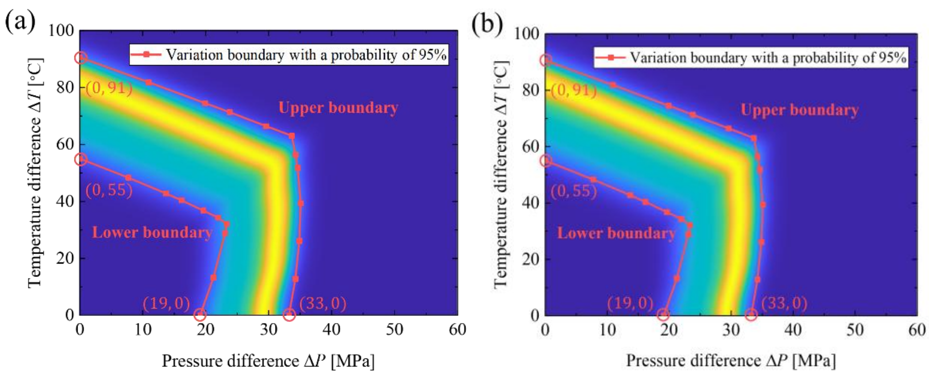


Figure 4. PDF surface for the stochastic shakedown limit obtained by (a) LMM-DPIM-NN model; (b) LMM-DPIM.

Influence of random parameters. The stochastic shakedown analysis results of PCHE structure under different random parameters are presented in Figure 5. The results shows that the fillet radius r at the channel corner exerts the most significant influence, in which its variation range induces a bimodal PDF and triggers stochastic P-bifurcation. The channel radius R also substantially affects the shakedown limit, while the thermal expansion coefficient α has the least influence.

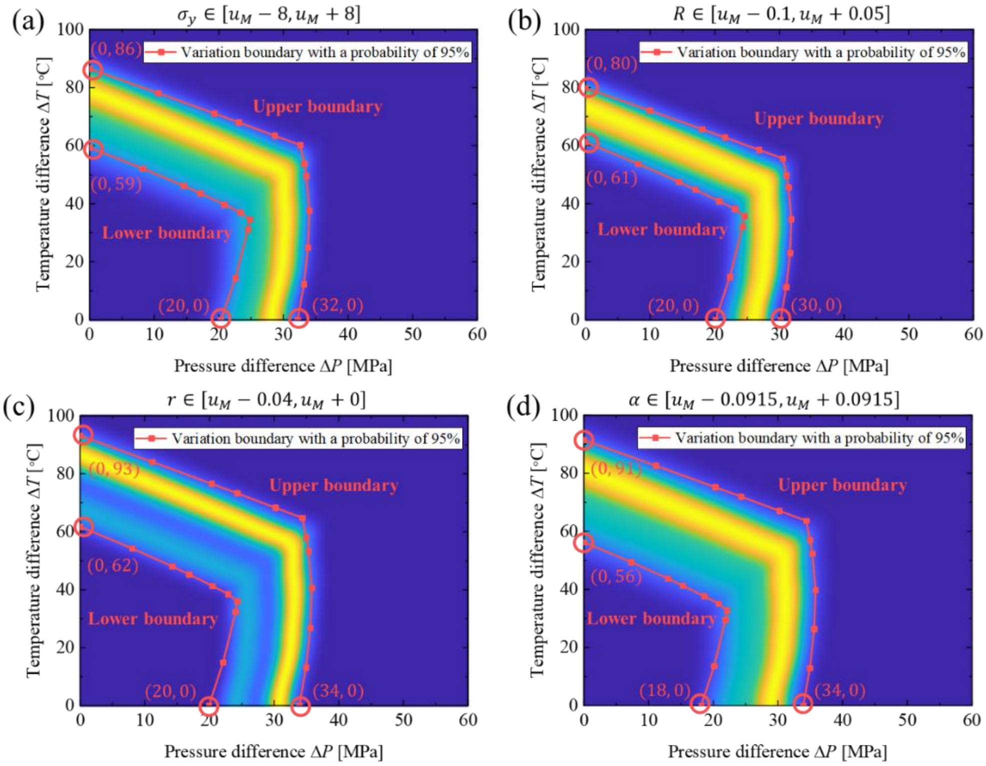
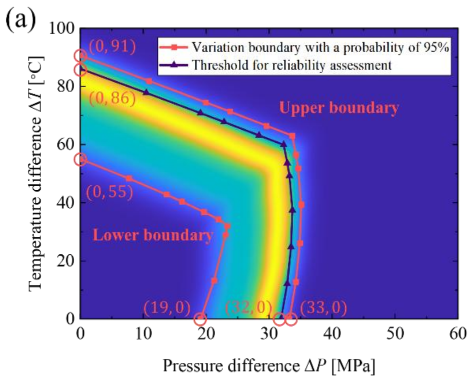


Figure 5. PDF surface for the stochastic shakedown limit for PCHE structure after the variations of different random parameters: (a) yield stress σ_y changed from [u_M−12, u_M+12] to [u_M−8, u_M+8]; (b) channel radius R changed from [u_M−0.1, u_M+0.25] to [u_M−0.1, u_M+0.05]; (c) fillet radius r changed from [u_M−0.08, u_M+0] to [u_M−0.04, u_M+0]; (d) thermal expansion α changed from [u_M−0.183, u_M+0.183] to [u_M−0.0915, u_M+0.0915].

Reliability analysis. Reliability curves are evaluated based on the shakedown limit threshold, as shown in Figure 6. Both R and r significantly affect reliability. The fillet radius r has the strongest impact: reliability decreases by 19.43% as r increases. The LMM-DPIM-NN reliability results agree well with MCS, validating the proposed framework for efficient stochastic reliability assessment.



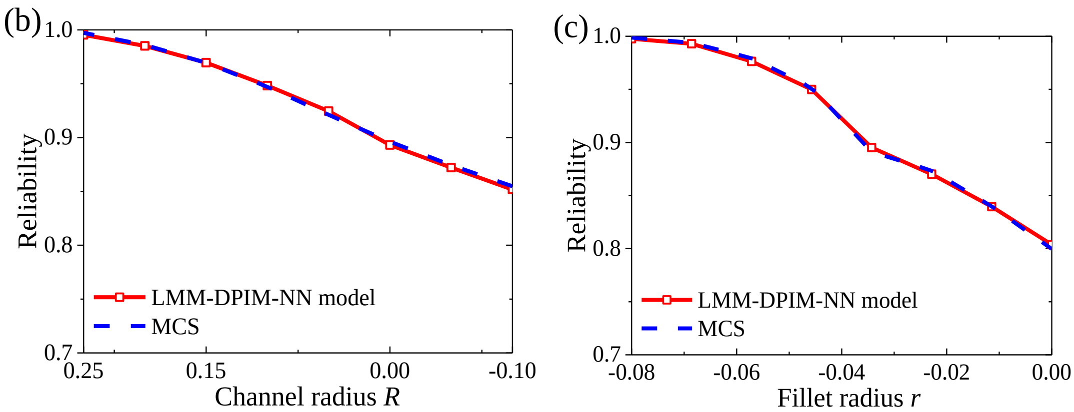


Figure 6. (a) Threshold for reliability assessment of PCHE structure; (b) reliability curve under different channel radius R; (c) reliability curve under different fillet radius r.

4. Conclusions

1. The LMM was combined with DPIM for the first time, and an LMM-DPIM-NN model was established for uncertainty qualification analysis of PCHEs with multiple random parameters. Verification against Monte Carlo simulation confirmed the accuracy of proposed method.

2. The LMM-DPIM-NN model achieves high accuracy comparable to LMM-DPIM while reducing CPU time, providing an efficient tool for stochastic shakedown analysis of PCHEs.

3. The fillet radius at the channel corner is the most influential random variable: it can induce stochastic P-bifurcation and causes reliability to decrease by up to 19.43 % as it increases, highlighting the importance of manufacturing tolerance control for this geometric parameter.

References

[1] Chen, M.H., X.D. Sun, R.N. Christensen, I. Skavdahl, V. Utgikar and P. Sabharwall. Pressure drop and heat transfer characteristics of a high-temperature printed circuit heat exchanger. *Applied Thermal Engineering*, 108: 1409–1417, 2016.

[2] Torre, R. de la., J. L. François and C. X. Lin. Assessment of the design effects on the structural performance of the Printed Circuit Heat Exchanger under very high temperature condition. *Nuclear Engineering and Design*, 365: 110713, 2020.

[3] Pham, D.C. Shakedown theory for elastic plastic kinematic hardening bodies. *International Journal of Plasticity*, 23: 1240–1259, 2007.

[4] *ASME Boiler and Pressure Vessel Code, Section III, Division 1—Subsection NH, in, The American Society of Mechanical Engineers*. New York, 2013.

[5] *R5: Assessment Procedure for the High Temperature Response of Structures*, in, EDF Energy.

[6] Bree, J. Elastic-plastic behaviour of thin tubes subjected to internal pressure and intermittent high-heat fluxes with application to fast-nuclear-reactor fuel elements, *Journal of strain analysis*, 2: 226–238, 1967.

[7] Polizzotto, C. On the conditions to prevent plastic shakedown of structures: Part I—Theory. *Journal of applied mechanics*, 60: 15–19, 1993.
[8] Peng, H., Y.H. Liu and H.F. Chen. A numerical formulation and algorithm for limit and shakedown analysis of large-scale elastoplastic structures. *Computational Mechanics*, 63: 1–22, 2019.
[9] Peng, H., Y.H. Liu, H.F. Chen and J. Shen, Shakedown analysis of engineering structures under multiple variable mechanical and thermal loads using the stress compensation method. *International Journal of Mechanical Sciences*, 140: 361–375, 2018.
[10] Chen, H.F., and A.R.S. Ponter. Shakedown and limit analyses for 3-D structures using the linear matching method. *International Journal of Pressure Vessels and Piping*, 78: 443–451, 2001.
[11] Chen, H.F., and A.R.S. Ponter. A method for the evaluation of a ratchet limit and the amplitude of plastic strain for bodies subjected to cyclic loading. *European Journal of Mechanics-A/Solids*, 20: 555–571, 2001.
[12] Koiter, W. T. *General theorems for elastic plastic solids*. Progress of Solid Mechanics, 1960.
[13] Barbera, D., H. Chen, Y. Liu, and F. Xuan. Recent developments of the linear matching method framework for structural integrity assessment. *Journal of Pressure Vessel Technology*, 139.5: 051101, 2017.
[14] Ma, Z.Y., Z.J. Fu, H.F. Chen and X.X. Wang. Shakedown and creep rupture analysis of printed circuit heat exchangers based on the linear matching method framework. *Finite Elements in Analysis and Design*, 233: 104127, 2024.
[15] Shen, J., Z.Y. Ma and H.F. Chen. Shakedown and ratcheting analysis of Printed Circuit Heat Exchangers under multiple cyclic mechanical and thermal loads. *International Journal of Pressure Vessels and Piping*, 199: 104723, 2022.
[16] Li, J. Probability density evolution method: background, significance and recent developments. *Probabilistic Engineering Mechanics*, 44: 111–117, 2016.
[17] Chen, G.H. and D.X. Yang. Direct probability integral method for stochastic response analysis of static and dynamic structural systems. *Computer Methods in Applied Mechanics and Engineering*, 357: 112612, 2019.
[18] Chen, H.S., Y.X. Gao, D.X. Yang, Z. Meng and Z.J. Fu. A novel weight index-based uniform partition technique of multi-dimensional probability space for structural uncertainty quantification. *Computer Methods in Applied Mechanics and Engineering*, 431: 117297, 2024.
[19] Wang, X.X., H.F. Chen and F.Z. Xuan. Direct method-based probabilistic shakedown analysis for the structure under multiple uncertain design conditions. *Ocean Engineering*, 280: 114653, 2023.
[20] Wang, R. Z., H. H. Gu, S. P. Zhu, K. S. Li, J. Wang, X. W. Wang and S. T. Tu. A data-driven roadmap for creep-fatigue reliability assessment and its implementation in low-pressure turbine disk at elevated temperatures. *Reliability Engineering & System Safety*, 225: 108523, 2022.
[21] Wang, X.X., J. Yang, H.F. Chen and F.Z. Xuan. Physics-based neural network for probabilistic low cycle fatigue and ratcheting assessments of pressurized elbow pipe component. *International Journal of Fatigue*, 172: 107598, 2023.
[22] Sepasdar, R., A. Karpatne and M. Shakiba. A data-driven approach to full-field nonlinear stress distribution and failure pattern prediction in composites using deep learning. *Computer Methods in Applied Mechanics and Engineering*, 397: 115126, 2022.